TWO-SECTOR MODEL WITH QUANTITY RATIONING

C. FOURGEAUD, B. LENCLUD AND P. MICHEL*

MARCH 1979

N° 7913

* UNIVERSITÉ DE PARIS I - CEPREMAP - DIRECTION DE LA PRÉVISION

MARCH 1979

TWO-SECTOR MODEL WITH QUANTITY RATIONING

C. FOURGEAUD, B. LENCLUD and P. MICHEL *

INTRODUCTION

The analysis of short-term economic equilibrium based on the adjustment of quantities, prices being fixed, has developed considerably in recent years. Parallel to formalizations in terms of general equilibrium and of reformulation of the microeconomic foundations of macroeconomy ([3], [6], [12], [14], ... see the survey in [7]), an aggregate macroeconomic model has been the object of systematic studies. Following CLOWER's article on the dual decision, BARRO and GROSSMAN [1] bring out the two situations involving excess supply and excess demand on the two markets (labour, goods) according to the level of prices and the level of wages. BENASSY [4] shows the possibility of a third area, that of classical unemployment. Various studies have developped the basic model ([2], [9], [11] ...) : MALINVAUD, in particular, emphasizes in the appendix to [11] the existence of a fourth area in case of stockage, an area dealt with in [13]. These analyses have made it possible to lay foundations for macroeconomic policies with greater precision, despite the schematic nature of the model, which comprises only one sector of production. The corresponding economic policies are overly rational and comforting. A wide variety of situations may exist in different sectors of the economy. This has prompted us to try and ascertain whether less clear-cut consequences appear in a model which distinguishes at least two sectors, those of consumer goods and investment goods.

---

* UNIVERSITE DE PARIS I - CEPREMAP - DIRECTION DE LA PREVISION. This work was supported in part by Direction de la Prévision, Ministère de l'Economie.

The authors are grateful to J.M. GRANDMONT and P.A. MUET, and other colleagues at CEPREMAP, for their helpful suggestions.

Our study present a two-sector model which takes into account both consumer and investment demand with differentiated behaviours in each sector. In addition to labour and money, the model comprises two final goods and one financial asset : bonds issued by the sector of production to finance their investments, and suscribed to by households. The investment demand of both sectors is dependent upon their level of activity, anticipation of outlets, the relative costs of capital and labour and the rate of interest. Public expenditures, financed by the creation of money, are both consumption and investment expenditures.

The logic of the model is essentially the logic of fixed-price models : the two prices of the goods and the wage rate are fixed. Conversely, the interest rate is the result of equilibrium between supply and demand on the securities market. Possible rationing in both sectors, on the investment goods and labour markets; is defined by a proportional scheme.

We shall prove the existence of an equilibrium for the system, analyzed according to the types of equilibrium prevailing in each of the two sectors : we thus obtain a typology by combining the various situations of partial equilibrium in each sector.

The variations in the parameters determining equilibrium are analyzed a simplified form of the preceding model. The consequences of certain economic policies are not as clear-cut as they appear to be in a single-sector model. For instance, in the Keynesian situation, there is a conflict between policies emphasizing an increase in output and those emphasizing employment. The impact of a variation in nominal wages on activity depends both on the sectors themselves and their type of equilibrium. This also applies to the effect of the parameters on the rate of interest, such that the final effect of certain measures appears to be highly uncertain.

The study is in three parts. Part I provides a description of the different elements of the model. In Part II, the different types of equilibrium and sufficient conditions for their existence are established. Part III analyzes the effects of policies and parameters on the general equilibrium of a simplified model in 3 situations : these being the two situations in which Keynesian and classical unemployment prevail, plus the third situation, where both sectors are in Keynesian unemployment.

I - PRESENTATION OF THE MODEL

I-1 The economy under consideration comprises :

- A capital good, price p_1 ;
- A consumer good, price p_2 ;
- Labour, nominal wage rate : w ;
- money, price $p_o = 1$;
- Bonds issued by the firms to finance their investments, and subscribed to by households (price q).

The four prices p_1, p_2, w and p_o are assumed to be fixed, the equilibrium is a short-term one achieved by quantity rationing. On the other hand, it is assumed (as it was assumed by Hool [10]) that the price q of the securities can vary on a market of the open-market type to balance supply and demand.

The four agents of the economy under consideration are :

- Households ;
- Government ;
- The capital goods sector of production (secteur 1) ;
- The consumer goods sector of production (sector 2).

Notation : equilibrium values are expressed in barred symbols.

I-2 The households's characteristics are :

- The initial stock of money : $M_{c,o}$;
- The initial stock of securities : B_o ;
- Their current incomes : $D_1(\overline{L}_1) + D_2(\overline{L}_2)$ which are the result of employment levels at equilibrium $\overline{L}_i$, in sector i(i = 1,2).

The labour supply L^o is assumed to be constant (prices being fixed).

By way of illustration, if the current incomes are wages, we have : $D_i(L_i) = wL_i$ (i = 1,2) ; but we may also consider the case of profits immediately distributed, at least in part (the case of individual entrepreneurs who use their profits to pay for what they consume).

Dividends from previous activities may appear in the initial stock of money, which is assumed to be net of the taxes,

./.

and includes interest on securities. However, expected income for subsequent periods (dividends, interest and value of securities, current income ...) must appear in the consumer's utility function :

(1) $$U = \alpha \text{Log.} C + \beta \text{Log.}(P+M_c) + \gamma \text{Log.}(Q+B)$$

with $\alpha, \beta, \gamma > 0$ and $\alpha + \beta + \gamma = 1$; we assume that P and Q are given <u>functions of the respective levels of activity of the two sectors</u> :

(2) $$P = P(L_1, L_2) \quad \text{et} \quad Q = Q(L_1, L_2)$$

The indirect utility of the money and securities can be obtained from an intertemporal model as in [13]. For example, by maximizing the utility of consumption over two periods : $\alpha \text{Log.} C_1 + \delta \text{Log.} C_2$ for wealth R_1 in the first period and expected income R_2^e for the second period, we have :

$$pC_1 + M_c + qB = R_1$$

$$p^e C_2 = R_2^e + M_c + q^e B + B$$

p^e and q^e being the expected prices of consumer goods and securities, respectively, for the second period ; according to whether q^e is lower or higher than $q - 1$, the totality of consumer savings will be in money or in securities ; and if we attribute to these two possibilities positive probabilities r and $1 - r$, the utility expectation (give or take one additive constant) is :

$$\alpha \text{Log.} C_1 + r\delta \text{Log.}(R_2^e + M_c) + (1-r)\delta \text{Log.}\left(\frac{R_2^e}{q^e+1} + B\right)$$

And we obtain expression (1), assuming that functions

$$P = R_2^e \quad \text{et} \quad Q = \frac{R_2^e}{q^e+1}$$

depend only upon the activity levels at equilibrium (and other constant parameters), i.e. do not depend directly upon the price of securities. We could dispense with this hypothesis, but that would complicate the problem, without significantly modifying the results obtained.

<u>REMARK</u> : The rather plausible condition $r < \frac{1}{2}$ (i.e. a greater probability of $\{q^e > q - 1\}$ than of $\{q^e < q - 1\}$ gives us $\beta < \gamma$.

The <u>budgetary constraint</u> of households is :

(3) $$p_2 C + M_c + \overline{q}\, B = R(\overline{L}_1, \overline{L}_2)$$

We shall see that for any employment levels L_1 and L_2 in the two sectors, we can derive from the equalité between the supply of and demand for securites, an equilibrium price $q(L_1, L_2)$. Thus, the wealth of households is associated with those levels of employment :

(4) $$R(L_1, L_2) = M_{c,o} + q(L_1, L_2)\, B_o + D_1(L_1) + D_2(L_2)$$

and we have, for the short-term equilibrium : $\overline{q} = q(\overline{L}_1, \overline{L}_2)$.

I-3 <u>The government</u>'s activities are confined to :

- a given consumption G ;
- a given investment I_G ;
- the creation of money corresponding to payment for its consumption and for its capital goods investment :

(5) $$\Delta M = p_1 I_G + p_2 G$$

Taxes may be collected outside of the period under consideration ; but the taxation rates, which do not play an explicit role in determining equilibrium, have had an impact on the monetary stocks of the other 3 agents, thereby affecting their behaviours, in particular consumers expectation P and Q and the investment demands in both sectors, as well as their monetary reserves. If need be, we can clearly define these influences.

I-4 <u>The consumer goods sector</u>

It is assumed that the aggregate consumer good cannot be stocked. The production function of sector 2, $F_2(L_2)$ depends upon its level of employment L_2, and upon a production capacity K_2.

./.

Its actual production, corresponding to level of employment L_2 is :

(6) $$Y_2 = \min \{F(L_2), K_2\}$$

This sector maximizes its profit

(7) $$\Pi_2(L_2) = p_2 F(L_2) - wL_2$$

subject to the following constraints :

(8) $$\begin{cases} \text{- production capacity : } F_2(L_2) \leq K_2 \\ \text{- employment : } L_2 \leq L^o - \bar{L}_1 \\ \text{- production outlets : } F(L_2) \leq C^d + G \end{cases}$$

C^d being the effective demand of households. We shall see that for any levels of employment L_1 and L_2 in the two sectors, the corresponding effective demand C^d (L_1, L_2) can be defined. The demand in sector 2 therefore is : C^d $(\bar{L}_1, L_2)$, in so far as production decisions of that sector affect the demand in that sector.

Sector 2 has a given initial stock of money $M_{2,o}$ which is assumed to be net of dividends and taxes related to its previous activities, and net of interest on the securities $B_{2,o}$. It decides to keep cash in hand : $M_2(L_2)$, a function of the employment level, it distributes income $D_2(L_2)$ (wages and part of the profits). The balance of its assets is :

(9) $$r_2(L_2) = p_2F_2(L_2) - D_2(L_2) + M_{2,o} - M_2(L_2)$$

These resources (which are generally positive) are completed by the issue or the redeeming of securities : $B_2 - B_{2,o}$, to finance its investment I_2 :

(10) $$p_1I_2 = r_2(L_2) + q(B_2 - B_{2,o})$$

./.

The desired investment is a function : 1/ of the levels of production in the two sectors, 2/ of the price of securities and, 3/ of various parameters which are constant in the determination of short-term equilibrium (production capacity K_2, prices p_2 and w, etc..). If we do not write the parameters out explicitly, we obtain function $A_2(L_1, L_2, q)$ which takes into account the demand $C^d(L_1, L_2) + G$, the expectations of the agent, etc...

The investment which is realized is conditioned by :

- a maximum limit of indebtedness

$$q(B_2 - B_{2,o}) \leq E_2(L_2)$$

The issue of securities is limited to a ceiling which is dependent upon the sector's level of production and the other parameters which are regarded as constant (in particular the volume of securities already issued).

- The limit to physical acquisitions possibilities :

$$I_2 \leq F_1(L_1) - I_G - I_1$$

It is assumed that in the event of quantity rationing of the capital good, the government is the first to be served, as is the case with the consumer good. Allocation of the capital good and of labor to the two sectors will be dealt with subsequently. The investment which is actually realized is therefore :

(11) $$I_2 = \min\{A_2(L_1,L_2,q), \frac{r_2(L_2)+E_2(L_2)}{p_1}, F_1(L_1) - I_G - I_1\}$$

and its demand for the capital good is :

(12) $$I_2^d = \min\{A_2(L_1,L_2,q), \frac{r_2(L_2)+E_2(L_2)}{p_1}\}$$

./.

I-5 The capital goods sector (sector 1).

Unlike sector 2, it is assumed that capital goods can be stocked. Sector 1 has an initial stock S_o, a production function $F_1(L_1)$ and a production capacity K_1. For a final stock S, the profit resulting from sales is :

$$\Pi_1(L_1,S) = p_1F_1(L_1) - p_1(S-S_o) - wL_1 \tag{13}$$

It is assumed that the firm maximizes a function which depends on the indirect utility of the stock resulting from an intertemporal optimization (as e.g. in [13]) :

$$V\ (\Pi_1(L_1,S),\ S) \tag{14}$$

subject to the following constraints :

$$\left\{\begin{array}{l} \text{- production capacity : } F_1(L_1) \le K_1 \\ \text{- employment : } L_1 \le L^o - \bar{L}_2 \\ \text{- stock : } S \ge 0 \\ \text{- production outlets, net of stock :} \\ \qquad F\ (L\ ) - (S-S_o) \le I_1^d + I_2^d + I_G \end{array}\right. \tag{15}$$

The investment desired, requested and realized in sector 1 are defined in the same way as in sector 2 :

$$\left\{\begin{array}{l} \qquad A_1(L_1,L_2,q) \\ I_1^d = \min\{A_1(L_1,L_2,q),\ \dfrac{r_1(L_1,S) + E_1(L_1)}{p_1}\} \\ \qquad I_1 = \min\{I_1^d,\ F_1(L_1) - I_G - I_2\} \end{array}\right. \tag{16}$$

where A_1 is a given function, $E_1(L_1)$ represents the maximum volume of securities issued :

./.

(17) $$q(B_1 - B_{1,o}) \leq E_1(L_1)$$

and $r_1(L_1,S)$ is the balance :

(18) $$r_1(L_1,S) = p_1F_1(L_1) - p_1(S-S_o) - D_1(L_1) + M_{1,o} - M_1(L_1)$$

The initial stock of money $M_{1,o}$ is given ; the final stock is defined by a function of the level of employment $M_1(L_1)$ and the immediately distributed income is $D_1(L_1)$. The investment realized is financed by the balance $r_1(L_1,S)$ and by the issuing (or withdrawal) of securities :

(19) $$p_1I_1 = r_1(L_1,S) + q(B_1-B_{1,o}).$$

The model is described completely, except for the effects of rationing.

I-6 Quantity rationing :

The securities markets is balanced, and subject to no constraints other than : $B_1 \geq 0$ and $B_2 \geq 0$.

The money market is also cleared . If we add the physical equilibrium equations, expressed in current prices to the budgetary equilibrium equations :

(20) $$\begin{cases} P_2(\overline{C} + G) = p_2F_2(L_2) \\ p_1(\overline{I}_1 + \overline{I}_2 + I_G + \Delta\overline{S}) = p_1\ F_1(\overline{L}_1) \\ p_1\ F_1(\overline{L}_1) + \overline{q}\ \Delta\overline{B}_1 = p_1\ \Delta\overline{S} + D_1(\overline{L}_1) + p_1\ \overline{I}_1 + \Delta\overline{M}_1 \\ p_2\ F_2(\overline{L}_2) + \overline{q}\ \Delta\overline{B}_2 = D_2(\overline{L}_2) + p_1\ \overline{I}_2 + \Delta\overline{M}_2 \\ D_1(\overline{L}_1) + D_2(\overline{L}_2) = p_2\overline{C} + \Delta\overline{M}_c + \overline{q}\ \Delta\overline{B} \end{cases}$$

./.

we obtain :

(21) $$p_2G + p_1I_G = \Delta\overline{M}_c + \Delta\overline{M}_1 + \Delta\overline{M}_2 = \Delta M$$

which is precisely the definition of government-created money. The only condition is that final stocks of money must not be negative.

On the consumer goods market, it is assumed that the government is the first to be served, so that :

(22) $$\overline{C} = \max\{0, F_2(\overline{L}_2) - G\}$$

On the capital good market, the demand stock is the first to be satisfied. Then the government is next to be served, and lastly the sectors are subjected to proportional rationing :

(23) $$\begin{cases} \overline{I}_1 + \overline{I}_2 = \max\{0, F_1(\overline{L}_1) - \Delta\overline{S} - I_G\} \\ \overline{I}_1 = k\, I_1^d \text{ et } I_2 = k\, I_2^d \, ; 0 \leq k \leq 1 \end{cases}$$

On the labour market, the two sectors are subjected to proportional rationing :

(24) $$\begin{cases} \overline{L}_1 + \overline{L}_2 = \overline{L} \leq L^0 \\ \overline{L}_1 = \ell\, L_1^d \text{ et } \overline{L}_2 = \ell\, L_2^d \, ; 0 < \ell \leq 1 \end{cases}$$

If we assume a great number of entreprises aggregated in each sector, proportional rationing between thetwo sectors seems reasonable.

./.

II - DETERMINATION OF EQUILIBRIUM

II-1 Equilibrium of the securities market

Let us consider the levels of activity of the two sectors as defined by levels of employment L_1 and L_2, respectively. At these levels, by virtue of the budgetary constraint of the government, the stock of money held by the households is :

(25) $$M_c(L_1L_2) = p_2G + p_1I_G + M_{c,o} + M_{1,o} + M_{2,o} - M_1(L_1) - M_2(L_2)$$

Let us study the case of values (L_1,L_2), where function $M_c(L_1,L_2)$ is strictly positive. Thus, at any level of consumption, since the consumer is not rationed on the monetary and bond markets, his demands for money M and for bonds B are solutions to the problem :

(26) $$\begin{cases} \text{maximum of } \beta \mathrm{Log}\,(P(L_1,L_2) + M_c) + \gamma \mathrm{Log}\,(Q(L_1,L_2) + B) \\ \text{subject to budgetary constraint : } M_c + qB = \text{constant} \end{cases}$$

Consequently, we obtain : $\dfrac{\gamma}{Q + B} = \dfrac{\beta q}{p + M_c}$, i.e.

(27) $$q(Q(L_1,L_2)+B) = \frac{\gamma}{\beta}\,(P(L_1,L_2) + M_c(L_1,L_2))$$

where B is the demand for securities expressed in terms of price q and levels L_1 and L_2.
The supply of securities in both sectors is defined by their budgetary equilibrium (relations (10) and (19)) ; by adding and taking into account the physical equilibrium of sector 1, we obtain :

(28) $$r_1(L_1,S)+r_2(L_2)+q(B-B_o) = p_1(I_1+I_2) = p_1(F_1(L_1) - I_G - \Delta S)$$

and substituting r_1 and r_2 with relations (9) and (18), the supply of securities is such that :

(29) $$q(B-B_o) = D_1(L_1)+M_1(L_1)-M_{1,o}-(p_2F_2(L_2)-D_2(L_2)-M_2(L_2)+M_{2,o})-p_1I_G$$

./.

By substracting (29) from (27), we obtain the equilibrium price of securities, which, taking (25) into account, is :

(30) $$q(L_1,L_2).\ (B_c+Q(L_1,L_2)) = \frac{\gamma}{\beta}(P(L_1,L_2)+p_2G+M_{c,o})+(\frac{\gamma}{\beta}+1)p_1I_G$$
$$-D_1(L_1)-(\frac{\gamma}{\beta}+1)(M_1(L_1)+M_2(L_2)-M_{1,o}-M_{2,o})+p_2F_2(L_2)-D_2(L_2)$$

This relation may also be expressed as a function of $M_c(L_1,L_2)$ (relation (25)) :

(31) $$q(L_1,L_2).(B_o+Q(L_1,L_2)) = \frac{\gamma}{\beta}(P(L_1,L_2)+M_c(L_1,L_2))-D_1(L_1)-D_2(L_2)$$
$$+ M_c(L_1,L_2)-M_{c,o}+p_2(F_2(L_2)-G)$$

The existence of an equilibrium price $q > 0$ results from assumptions about income expectations P, so thatthe right-hand side of equation (13) is positive. With $\gamma \geq \beta$ (cf. § I.2 remark), these conditions are fulfilled if the sum of expected income P and monetary savings exceeds the share of current income assigned to additional purchases of bonds : $q\Delta B = D_1 + D_2 - \Delta M_c - p_2C$.

The equilibrium level of the bonds is then the solution of :

$$[\frac{\gamma}{\beta}(P+M_c)-D_1-D_2+\Delta M_c+p_2C]\ (B+Q) = \frac{\gamma}{\beta}(P+M_c)\ (B_o+Q)$$

and B is positive if expectations P and Q are such that :

$$\frac{\gamma}{\beta}(P+M_c)B_o+Q(D_1+D_2-\Delta M_c-p_2C) > 0$$

II-2 The effective demand of the consumer.

For given levels of employment L_1 and L_2, "the consumer" maximizes his utility function (1) subject to his budget constraint :

$$p_2C+M_c+qB = M_{c,o}+qB_o+D_1(L_1)+D_2(L_2) = R(L_1,L_2)$$

./.

Being unrestricted, on both the monetary market and the bond market, his effective demand for the consumer good is :

(32) $$C^d(L_1,L_2) = \frac{\alpha}{p_2}\,(P(L_1,L_2)+q(L_1,L_2)Q(L_1,L_2)+R(L_1,L_2))$$

and with (31), we obtain :

(33) $$C^d(L_1,L_2) = \frac{\alpha}{p_2}\,(p_2(F_2(L_2)-G)+(\frac{\gamma}{\beta}+1)(P(L_1,L_2)+M_c(L_1,L_2))$$

REMARK

We note that the effective consumption demand ultimately depends upon the activity of sector 1 only through money and expectations of future income. An increase employment, and hence in wages paid out, in the investment goods sector, induces the financing of additional investment : most of the additional income of households is used for the purchase of securities needed for this financing. The value of these securities then decreases (relation (30)), which corresponds to an increase in the interest rate.

II-3 Equilibrium of the Consumer Goods Sector.

For a given level of employment L_1, in sector 1, sector 2 maximizes its profit (7) under constraints (8) for production capacity, labour supply and production outlets.

Assumption 1 : the production function $F_2(L_2)$ is such that :

$$F_2' > 0,\ F_2'' < 0,\ F_2(0) = 0,\ F_2(\infty) = \infty,\ F_2'(0) = \infty \text{ et } F_2'(\infty) = 0$$

Assumption 2 : the function $F_2(L_2) - C^d(L_1,L_2)$ is for a fixed L_1, an increasing function of L_2 in the increasing profits area.

The first hypothesis is the usual one and corresponds to diminishing returns. The second hypothesis expresses the fact that in relation

./.

to the level of employment, consumption demand increases less rapidly than production does. In our model, this hypothesis holds if P is independant of L_2, because then we have :

$$F_2(L_2) - C^d(L_1,L_2) = (1 - \alpha)F_2(L_2) + M_2(L_2) + \text{constant}.$$

Assumption 2 therefore implies a condition upon the increase in expectations P according to the level of employment.

The following levels are defined for sector 2 :

- level $L_2^o(L_1)$ of full employment of labour : $L_2^o(L_1) = L^o - L_1$;
- level of employemnt L_2^K corresponding to production capacity : $F_2(L_2^K) = K_2$;
- level of employment L_2^2 corresponding to the maximum profit : $F_2'(L_2^2) = \frac{w}{p_2}$;
- level of employment $L_2^3(L_1)$ corresponding to the satisfaction of demand : $F_2(L_2^3(L_1)) = G + C^d(L_1,L_2^3(L_1))$.

Thus, as a result of the hypotheses for a given L_1, the equilibrium of the sector of the consumer good corresponds to the level of employment :

(34) $$\hat{L}_2(L_1) = \min \{L_2^o(L_1), L_2^K, L_2^2, L_2^3(L_1)\}$$

and its demand for labour is :

(35) $$L_2^d(L_1) = \min \{L_2^K, L_2^2, L_2^3(L_1)\}$$

<u>REMARK 1</u>

Without any special assumption, the optimal solution(s) to sector 2's program is (are) a function $\hat{L}_2(L_1)$ and the solution to the program without the employment supply constraint is a function

./.

$L_2^d(L_1)$. Assuming that these solutions exist and are unique, it is possible to determine the short-run equilibrium of the economy ; if there is no uniqueness, the same method applies to the different solutions which define different equilibria.

REMARK 2

With the above hypotheses, expression (34) enables us to define 4 major types of equilibrium for sector 2 :

- repressed inflation for $\hat{L}_2 = L_2^o$
- unemployment due to an insufficient production capacity for $\hat{L}_2 = L_2^K$
- classical unemployment for $\hat{L}_2 = L_2^2$
- Keynesian unemployment for $\hat{L}_2 = L_2^3$

II-4 The equilibrium of the capital goods sector when demand for labour in both sectors does not exceed supply.

For an employment level L_1 in sector 1, the investment demand in sector 2 corresponding to its equilibrium $\hat{L}_2(L_1)$ is derived from the relations (12) and (31) :

(36) $$\hat{I}_2^d(L_1) = \min\{A_2(L_1,\hat{L}_2(L_1),q(L_1,\hat{L}_2(L_1)),\frac{1}{p_1}(r_2(\hat{L}_2(L_1))+E_2(\hat{L}_2(L_1)))\}$$

Let us assume :

(37) $$\begin{cases} I^d(L_1) = \hat{I}_2^d(L_1)+A_1(L_1,\hat{L}_2(L_1),\ q(L_1,\hat{L}_2(L_1))) + I_G \\ J^d(L_1,S) = \hat{I}_2^d(L_1) + \frac{1}{p_1}(r_1(L_1,S) + E_1(L_1)) + I_G \end{cases}$$

Then the total demand for investment is :

(38) $$I^d = \min\{I^d(L_1),\ J^d(L_1,S)\}$$

./.

The production outlet constraint in sector 1 is equivalent to the following conditions (39) :

$$(39)\qquad \begin{cases} F_1(L_1) - (S-S_o) \le I^d(L_1) \\ F_1(L_1) - (S-S_o) \le J^d(L_1,S) \end{cases}$$

The second inequality in (39) may be expressed independently of S ; substituting J^d and r_1 their respective definitions (37) and (18), we obtain :

$$(40)\qquad D_1(L_1) + M_1(L_1) \le M_{1,o} + E_1(L_1) + p_1(\hat{I}_2^d(L_1) + I_G)$$

The demand for employment L_1^d and the corresponding stock S^d of sector 1 are the solution to the program :

$$\max \varphi(\Pi_1(L_1,S),S)$$

$$(41)\qquad \begin{cases} F_1(L_1) \le K \\ S \ge 0 \\ F_1(L_1) - (S-S_o) \le I^d(L_1) \\ D_1(L_1) + M_1(L_1) \le M_{1,o} + E_1(L_1) + p_1(\hat{I}_2^d(L_1) + I_G). \end{cases}$$

Consequently, if solution L_1^d to this program is such that : $L_1^d + L_2^d\,(L_1^d) \le L°$, the equilibrium of sector 1 is $\overline{L}_1 = L_1^d$ and $\overline{S} = S^d$. We can make this solution more explicit by making certain assumptions. We name $S^1(L_1)$ the solution (assumed to be unique) to : maximum of $\varphi(\Pi_1(L_1,S),S)$ for $(S \ge 0)$, L_1 being fixed.
If the function $\varphi(\Pi_1(L_1,S),S)$ is decreasing with respect to S for $S > S^1(L_1)$, then the optimal level of the stock in relation to employment L_1 is :

$$(42)\qquad S(L_1) = \max\,\{S^1(L_1),S^2(L_1)\}\ ,\ S^2(L_1) = F_1(L_1) - I^d(L_1) + S_o$$

./.

Assuming that :

- for a fixed L_1, $\varphi\,(\Pi_1(L_1,S),S)$ is decreasing with respect to S for $S > S^1(L_1)$ and reaches its maximum at $S^1(L_1)$;
- $F_1(L_1)$ is increasing and reaches K_1 for L_1^K ;
- $\varphi\,(\Pi_1(L_1,S(L_1)),S(L_1))$ is increasing with respect to L_1 for $L_1 < L_1^2$ and reaches its maximum at L_1^2 ;
- $D_1(L_1)+M_1(L_1)-E_1(L_1)-p_1\hat{I}_2^d(L_1)$ is increasing with respect to L_1 (at least for $L_1 < L_1^2$) and reaches $M_{1,o}+p_1I_G$ for L_1^3 ;

then the demand for labour L_1^d is defined by :

(43) $$L_1^d = \min\,\{L_1^K,\ L_1^2,\ L_1^3\}$$

We obtain the three types of unemployment (L_1^K due to production capacity, L_1^2 classical and L_1^3 Keynesian) ; the stockage is either constrained, or not, according to whether $S^2(L_1)$ is larger or smaller than $S^1(L_1)$.

II-5 General equilibrium.

When the demand for employment does not exceed the supply, that is if $L_1^d + L_2^d(L_1^d) \leq L°$, then the equilibrium is defined by

(44) $$\overline{L}_1 = L_1^d\ ,\ \overline{L}_2 = L_2^d(\overline{L}_1) = \hat{L}_2(\overline{L}_1)\ \text{et}\ \overline{S} = S(\overline{L}_1).$$

There are 9 main types of equilibrium according to whether each of the two sectors is in a situation of unemployment :
1/ due to insufficient production capacity, 2/ of classical unemployment or 3/ of Keynesian unemployment.

When demand for employment exceeds supply, there exists a positive number $a < 1$ such that $aL_1^d + aL_2^d(aL_1^d) = L°$; and it is unique

./.

if $L_2^d(L_1)$ is not decreasing (and consequently if $L_2^3(L_1)$ is not decreasing). Then the equilibrium is defined by :

(45) $$\overline{L}_1 = aL_1^d \text{ , } \overline{S} = S(\overline{L}_1) \text{ and } \overline{L}_2 = \hat{L}_2(\overline{L}_1) = L° - \overline{L}_1$$

This equilibrium is of a repressed inflation type for both sectors ; it is of one of two types, according to whether the supply of or demand for, capital goods is constrained ; i.e. according to whether $S^1(\overline{L}_1)$ is larger or smaller than $S^2(\overline{L}_1)$. In every case where the damand for capital goods is constrained, the available production $F_1(\overline{L}_1) - (\overline{S} - S_o) - I_G$ is distributed proportionally to demands I_1^d and I_2^d.

III - EFFECTS ON THE EQUILIBRIUM OF THE PARAMETERS

In this section we shall consider the effects, on the equilibrium, of variations in the parameters such as : public expenditure (consumption and investment), prices and wages.

To do this, we shall consider a simplified form of the model.

III-1 Simplified form of the model

We assume there is no stockage in sector 1, and that current income only consists of wages :

(46) $$D_1(L_1) = wL_1 \quad \text{et} \quad D_2(L_2) = wL_2$$

To simplifiy the study, we shall assume that the functions P, Q, M_1, M_2, A_1 and A_2 (expectations of households, cash-in-hand of firms, desired investments) do not explicitly depend on L_1 and L_2. Specifically, the desired investment depends only on

./.

the rate of interest $\frac{1}{q}$, on expectations of outlets, on the production capacity and on the price system. This is consistent with the usual formalizations of investment functions which depend on both the cost of using capital and expectations.

If we introduce these assumptions into the equations of the general model, we obtain :

$$(47)\quad \begin{cases} M_c = p_2G + p_1I_G + M_{c,o} - \Delta M_E \\ q(B_o+Q) = \frac{\gamma}{\beta}(P+p_2G+M_{c,o})+(\frac{\gamma}{\beta}+1)(p_1I_G-\Delta M_E)+p_2F_2(L_2)-wL_1-wL_2 \\ C^d(L_1,L_2) = \frac{\alpha}{p_2}\left[p_2(F_2(L_2)-G)+(\frac{\gamma}{\beta}+1)(P+M_c)\right] \end{cases}$$

where $\Delta M_E = M_1 - M_{1,0} + M_2 - M_{2,0}$

We deduce from (47) :

$$(48)\quad F_2(L_2)-C^d(L_1,L_2) = (1-\alpha)F_2(L_2)-\frac{\alpha\gamma G}{\beta}-\frac{\alpha(1-\alpha)}{\beta p_2}(P+M_{c,o}-\Delta M_E+p_1I_G)$$

which is an increasing function of L_2 ; the demand for labour L_2^d of sector 2 is therefore defined by :

$$(49)\quad \begin{cases} L_2^d = \min\{L_2^K,\ L_2^2,\ L_2^3\} \\ L_2^K = F_2^{-1}(K_2) \\ L_2^2 = F_2'^{-1}(\frac{w}{p_2}) \\ L_2^3 = F_2^{-1}((\frac{\alpha+\beta}{\beta})G+\frac{\alpha}{\beta p_2}(P+M_{c,o}-\Delta M_E+p_1I_G)) \end{cases}$$

Similarly, with the assumptions made for the general model (§ 2.4), we have for sector 1 :

./.

$$(50) \quad \begin{cases} L_1^d = \min \{L_1^K,\ L_1^2,\ L_1^3\} \\ L_1^K = F_1^{-1}(K_1) \\ L_1^2 = F_1'^{-1}(\frac{w}{p_1}) \\ L_1^3 = F_1^{-1}(\hat{I}_1^d + \hat{I}_2^d + I_G) \end{cases}$$

III-2 Definition of the framework of the study

Given the multiplicity of types of equilibrium, we shall select three types for our study :

A) Sector 1 with classical unemployment and sector 2 with Keynesian unemployment.

B) Sector 1 with Keynesian unemployment ans sector 2 with classical unemployment.

C) Both sectors with Keynesian unemployment.

Furthermore, we shall limit our study to the case in which the demands for investment in both sectors are not restricted by indebtedness constraints $E_i(L_i)$. In the selected types of equilibrium, the demands for labour in the sectors are satisfied and the demands for investment are as follows, according to the assumptions already made :

$$(51) \quad \hat{I}_1^d = A_1(q(L_1,L_2),V) \quad \text{et} \quad \hat{I}_2^d = A_2(q(L_1,L_2),V)$$

V stands for the other non explicit parameters : prices, wages, expectations.

Later, we shall be led to make use of assumptions about the investment functions, the demand of the sectors for money, the incidence of prices and wages on the expectations of households.

./.

. Assumptions about the investment functions :

$$\frac{\partial A_i}{\partial q} > 0, \ \frac{\partial A_i}{\partial w} > 0, \ \frac{\partial A_i}{\partial p_1} < 0 \qquad (i = 1, 2)$$

Such assumptions express the fact that the demand for investment in each sector is a decreasing function of the interest rate $(\frac{1}{q})$ as well as a decreasing function of the relative capital-labour cost.

. Assumptions about the demand of sectors for money :

$$\frac{\partial M_1}{\partial p_1} > 0, \ \frac{\partial M_2}{\partial p_2} > 0 \ ; \ \frac{\partial M_1}{\partial w} < 0, \ \frac{\partial M_2}{\partial w} < 0$$

Money is, for the most part, destined for the payment of taxes and distribution of profits. All other things being equal, we may assume that they vary directly with the production price and inversely with the wage rate.

. Assumptions about the income expectations of households :

$$\frac{\partial P}{\partial w} > 0, \ \frac{\partial P}{\partial p_2} > 0$$

The assumptions express the fact that expectations on nominal income are an increasing function of the nominal wage rate and of the consumer price.

III-3 <u>Study of the case of classical unemployment in sector 1 and Keynesian unemployment in sector 2</u>

This case is characterized by :

$$(52) \qquad \begin{cases} F_1'(L_1) = \dfrac{w}{p_1} \\[2ex] F_2(L_2) = (1+\dfrac{\alpha}{\beta})G + \dfrac{\alpha}{\beta p_2}(N+p_1 I_G) \end{cases}$$

where $N = P + M_{c,o} + M_{1,o} - M_1 + M_{2,o} - M_2$

./.

We thus obtain :

(53)
$$\begin{cases} F_1''(L_1)dL_1 = \frac{1}{p_1}dw - \frac{w}{p_1^2}dp_1 \\ F_2'(L_2)dL_2 = (1+\frac{\alpha}{\beta})dG - \frac{\alpha}{\beta p_2^2}(N+p_1 I_G)dp_2 + \frac{\alpha}{\beta p_2}(dN+p_1 dI_G+I_G dp_1) \end{cases}$$

a) Effect of public consumption G :

(54)
$$dY_2 = F_2'(L_2)dL_2 = (1+\frac{\alpha}{\beta})dG + \frac{\alpha}{\beta p_2}\frac{\partial N}{\partial G}dG$$

With no effect on the production of sector 1, government consumption has a stimulating effect on sector 2 with the usual Keynesian multiplier $1+\frac{\alpha}{\beta}$, modified by possible effects on the expectation of households and firms liquid assets.

b) Effect of public investment I_G :

assuming that $\frac{\partial N}{\partial I_G}$ is negligible, we obtain :

(55)
$$dY_2 = F_2'(L_2)dL_2 = \frac{\alpha p_1}{\beta p_2}dI_G$$

Since $\alpha > \beta$, the effect of nominal public investment :

$$\frac{p_2 dY_2}{p_1 dI_G} = \frac{\alpha}{\beta} > 1$$

is a multiplier effect, but :

- Such an effect in smaller than that of public consumption, for an equal level of expenditure :

$$\frac{p_2 dY_2}{p_2 dG} = 1 + \frac{\alpha}{\beta}$$

This policy reduces to the same degree rationed private investments, since the total production of capital goods does not depend on demand.

./.

c) Effect of a change in the wage rate w :

Other prices being constant, we have :

(56)
$$\begin{cases} F''(L_1)dL_1 = \dfrac{1}{p_1}\,dw \\ F_2'(L_2)dL_2 = \dfrac{\alpha}{\beta p_2}\,\dfrac{\partial N}{\partial w}\,dw \end{cases}$$

(57)
$$\frac{\partial N}{\partial w} = \frac{\partial P}{\partial w} - \frac{\partial M_1}{\partial w} - \frac{\partial M_2}{\partial w}$$

Under the assumptions set forth, if prices p_1 and p_2 remain constant, the increase in the nominal wage rate w increases the income expectations of households and is likely to have a negative effect on the monetary reserves of firms.

The increase in w then has :

. a stimulating effect on sector 2 through its effects on the income expectations of households :

$$\frac{dY_2}{dw} = \frac{\alpha}{\beta p_2}\,\frac{\partial N}{\partial w} > 0$$

. a depressing effect on sector 1 through the decrease in profit :

$$\frac{dY_1}{dw} = \frac{F_1'(L_1)}{p_1 F_1''(L_1)} < 0$$

d) Effects of a change in the price p_1 of capital goods :

(58)
$$\begin{cases} F_1''(L_1)dL_1 = -\dfrac{w}{p_1^2}\,dp_1 \\ F_2'(L_2)dL_2 = \dfrac{\alpha}{\beta p_2}\left(\dfrac{\partial N}{\partial p_1} + I_G\right)dp_1 \end{cases}$$

The increase in p_1 has :

. a stimulating effect on sector 1 through the increase in its profit :

$$\frac{dY_1}{dp_1} = -\frac{w}{p_1^2}\,\frac{F_1'(L_1)}{F_1''(L_1)} > 0$$

./.

. an effect on sector 2 made up of two terms :

1) $\frac{\alpha}{\beta p_2} I_G > 0$ corresponding to the additional creation of money by the government to pay for its investments ;

2) $\frac{\alpha}{\beta p_2} \frac{\partial N}{\partial p_1} \simeq - \frac{\alpha}{\beta p_2} \frac{\partial M_1}{\partial p_1}$; assuming that the most important effect of p_1 is felt by the monetary reserves of sector 1 and that these reserves are increasing along p_1, we obtain a negative effect.

Consequently, the resulting effect on sector 2 may be positive or negative. It depends on whether the additional government expenditures resulting from the increase in the price p_1 exceed or do not exceed the additional monetary reserves decided upon by sector 1. In the medium run, this effect will in any case be positive on account of the increase in the income distributed by sector 1.

e) <u>Changes in the price</u> p_2 <u>of consumer goods</u> :

These changes have no effect on the first sector. For the second sector, we have :

(59) $$F_2'(L_2)dL_2 = - \frac{\alpha}{\beta p_2^2} (N+p_1 I_G)dp_2 + \frac{\alpha}{\beta p_2} \frac{\partial N}{\partial p_2} dp_2$$

(60) $$\frac{\partial N}{\partial p_2} \simeq \frac{\partial P}{\partial p_2} - \frac{\partial M_2}{\partial p_2}$$

According to our assumptions, the increase in p_2 has a positive effect both on the income expectations of households and on the decisions concerning monetary reserves made by the firms in sector 2. The sign of $\frac{\partial N}{\partial p_2}$ is incertain, but we may think that in (59) the first term is dominant, such that an increase in the consumer price exerts a depressing effect on sector 2.

./.

In short, we have the following effects :

	G	I_G	w	p_1	p_2
Sector 1	0	0	--	++	0
Sector 2	++	+	+	?	-

REMARK : As sector 1 is in a classical unemployment situation, the demand for investment exceeds supply and only the wage rate and the price p_1 are apt to modify its level of activity.

III-4 Study of the case of Keynesian unemployment in sector 1 and of classical unemployment in sector 2.

This case is characterized by :

(61)
$$\begin{cases} F_1(L_1) = I_G+A_1(q,V)+A_2(q,V) = I_G+A(q,V) \\ F_2'(L_2) = \dfrac{w}{p_2} \end{cases}$$

We thus obtain :

(62)
$$\begin{cases} dY_1 = F_1'(L_1)dL_1 = dI_G + \dfrac{\partial A}{\partial q}dq + \dfrac{\partial A}{\partial V}dV \\ F_2''(L_2)dL_2 = \dfrac{1}{p_2}dw - \dfrac{w}{p_2^2}dp_2 \end{cases}$$

The variation in q is derived from (47) :

(63)
$$(B_o+Q)dq + qdQ = \frac{\gamma}{\beta}(dP+p_2dG+Gdp_2) + (\frac{\gamma}{\beta}+1)(p_1dI_G+I_Gdp_1)$$
$$+ p_2F_2'(L_2)dL_2+F_2(L_2)dp_2-w(dL_1+dL_2) - (L_1+L_2)dw$$

./.

a) <u>Effect of public consumption G</u> :

No effect on sector 2 ; in sector 1 we have :

(64) $$dY_1 = \frac{\partial A}{\partial q} \frac{1}{(B_o+Q)} (\frac{\alpha}{\beta} p_2 dG - wdL_1)$$

Let : $k = \frac{A}{q} \frac{1}{B_o+Q}$ et $\frac{1}{\delta} = 1 + \frac{kw}{F_1'(L_1)}$

Under the assumptions, k is positive and we therefore have : $0 < \delta < 1$. Such that (64) becomes :

(65) $$dY_1 = k\delta \frac{\gamma}{\beta} p_2 dG$$

b) <u>Effect of public investment</u> I_G :

. No effect on sector 2.

. On sector 1 we have :

(66) $$dY_1 = dI_G + \frac{\partial A}{\partial q} \frac{1}{B_o+Q} ((\frac{\gamma}{\beta} + 1)p_1 dI_G - wdL_1)$$

and we obtain :

(67) $$dY_1 = \delta(1 + k(\frac{\gamma}{\beta} + 1)p_1)dI_G$$

<u>REMARK</u> : Given the same increase in the public expenditures ($p_2dG = p_1dI_G$) it is obviously the investment expenditures which have the greater effect on the activity in sector 1. For public consumption expenditures, the increase in activity results from the effect on investment of a reduction in the interest rate ($dq > 0$). This investment can be financed by an increase in "unvoluntary savings" of households (whose consumption decreases by dG) enabling them to subscribe to additional securities issued by the industrial sectors.

Employment varies directly to activity.

c) Effect of the wage rate :

Other prices being constant, we have :

(68)
$$\begin{cases} F''(L_2)dL_2 = \dfrac{1}{p_2}\, dw \\ F_1'(L_1)dL_1 = \dfrac{\partial A}{\partial w}\, dw + \dfrac{\partial A}{\partial q}\dfrac{1}{B_o+Q}\,[p_2F_2'(L_2)dL_2 - w(dL_1+dL_2) - (L_1+L_2)dw] \end{cases}$$

We obtain, with $p_2F_2'(L_2) = w$,

(69)
$$\begin{cases} dY_1 = F_1'(L_1)dL_1 = \delta\, [\dfrac{\partial A}{\partial w} - k(L_1+L_2)]dw \\ dY_2 = \dfrac{F_2'(L_2)}{p_2F_2''(L_2)}\, dw \end{cases}$$

by substituting the corresponding terms from (69) in the expression of dq for the variation dL_1, we obtain :

(70)
$$dq = \frac{-\,\delta}{B_o+Q}\left[\frac{w}{F_1'(L_1)}\frac{\partial A}{\partial w} + (L_1+L_2)\right]dw$$

A rise in nominal wages depresses the consumer goods sector through a reduction of the protitable production capacity. The effect is more questionable for the investment goods sector. Indeed, $\frac{\partial A}{\partial w}$ is positive owing to the relative decrease in capital cost ; but, on the other hand, according to (70), the interest rate rises ($dq < 0$). In expression (69) of dY_1, we may assume that the second effect prevails over the first and therefore, in the aggregate, a rise in nominal wages is in this case conducive to a reduction in activity and employment.

d) Effect of the variations of p_1 :

(71)
$$\begin{cases} dY_2 = 0 \\ F_1'(L_1)dL_1 = \dfrac{\partial A}{\partial p_1}\, dp_1 + k\,[(\dfrac{\gamma}{\beta}+1)I_G dp_1 - wdL_1] \end{cases}$$

./.

(72) $$dY_1 = \delta \left[\frac{\partial A}{\partial p_1} + k(\frac{\gamma}{\beta} + 1)I_G\right] dp_1$$

Substituting the corresponding terms from (72) in the expression of dq for the variation of L_1, we obtain :

(73) $$dq = \frac{\delta}{B_o+Q} \left[(\frac{\gamma}{\beta} + 1)I_G - \frac{w}{F_1'} \frac{\partial A}{\partial p_1}\right] dp_1$$

We have : $\frac{\partial A}{\partial p_1} < 0$ since an increase in capital cost lowers the investment demand. Furthermore, public expenditures for investments rises by $I_G dp_1$. The second term in (72) therefore represents a multiplier effect similar to an increase in I_G (equation (67)). The resulting effect is indeterminate. In expression (75) of the variation of q, we see that the rate of interest decreases. The impact on the investment demand is thus the inverse of an increase in p_1 and, without a more precise specification of investment functions, the aggregate effect cannot be estimated.

e) <u>Influence of the variations in</u> p_2 :

(74) $$\begin{cases} F_2''(L_2)dL_2 = -\frac{w}{p_2^2} dp_2 \\ F_1'(L_1)dL_1 = \frac{\partial A}{\partial p_2} dp_2 + k[\frac{\gamma}{\beta} Gdp_2 - wdL_1] \end{cases}$$

(75) $$\begin{cases} dY_1 = \delta[\frac{\partial A}{\partial p_2} + k \frac{\gamma}{\beta} Gdp_2] \\ dY_2 = -\frac{wF_2'(L_2)}{p_2^2 F_2''(L_2)} dp_2 = -\frac{w^2}{p_2^3 F_2''(L_2)} dp_2 \end{cases}$$

The rise in prices in the consumer goods sector increases the profitable production capacity, in this sector and consequently increases employment. We may consider that function A, if dependent on p_2, is such that $\frac{\partial A}{\partial p_2}$ is positive, so that the activity in the investment goods sector also increases.

./.

In short, we have the following effects :

	G	I_G	w	p_1	p_2
Sector 1	+	++	-	?	+
Sector 2	0	0	-	0	+

III-5 Study of the case of Keynesian unemployment in both sectors.

This case is defined by :

(76)
$$\begin{cases} F_1(L_1) = I_G + A_1(q,V) + A_2(q,V) = I_G + A(q,V) \\ F_2(L_2) = \dfrac{\alpha+\beta}{\beta} G + \dfrac{\alpha}{\beta p_2}(N + p_1 I_G) \end{cases}$$

with $N = P + M_{c,o} + M_{1,o} - M_1 + M_{2,o} - M_2$;

we thus obtain :

(77)
$$\begin{cases} F'_1(L_1)dL_1 = dI_G + \dfrac{\partial A}{\partial q} dq + \dfrac{\partial A}{\partial V} dV \\ F'_2(L_2)dL_2 = \dfrac{\alpha+\beta}{\beta} dG + \dfrac{\alpha}{\beta p_2}(\dfrac{\partial N}{\partial V} dV + p_1 dI_G + I_G dp_1) - \dfrac{\alpha}{\beta p_2^2}(N + p_1 I_G)dp_2 \end{cases}$$

The variation fo q is derived from (47)

$$(B_o+Q)dq + qdQ = \frac{\gamma}{\beta}(dP+p_2dG+Gdp_2) + (\frac{\gamma}{\beta}+1)(p_1dI_G + I_Gdp_1) + p_2F'_2(L_2)dL_2$$
$$+ F_2(L_2)dp_2 - w(dL_1 + dL_2) - (L_1 + L_2)dw$$

The effect of a public consumption and investment variation on N is considered to be negligible.

./.

a) <u>Effect of public consumption G</u> :

The effect on sector 2 is expressed by

(78) $$dY_2 = F_2'(L_2)dL_2 = (1 + \frac{\alpha}{\beta})dG$$

$1 + \frac{\alpha}{\beta}$ is the usual Keynesian multiplier.

The effect on sector 1 is defined by :

(79) $$dY_1 = \frac{\partial A}{\partial q} \frac{1}{B_o + Q} \left[\frac{\gamma}{\beta} p_2 dG - w dL_1 + (p_2 F_2'(L_2) - w) dL_2\right]$$

Let : $$\begin{cases} k = \dfrac{1}{B_o + Q} \dfrac{\partial A}{\partial q} \\ \dfrac{1}{\delta} = 1 + k \dfrac{w}{F_1'(L_1)} \\ \text{et} \quad \sigma_i = \dfrac{p_i F_i'(L_i) - w}{p_i F_i'(L_i)} \qquad (i = 1,2) \end{cases}$$

σ_i is the marginal profit rate in sector i.

We obtain :

(80) $$dY_1 = F_1'(L_1)dL_1 = k\delta p_2 \left[\frac{\gamma}{\beta} + (1 + \frac{\alpha}{\beta})\sigma_2\right] dG$$

Let us now examine the aggregate effect of an increase dG on the volume of production and employment.

. The μ_G multiplier of aggregate production is defined by

$$p_1 dY_1 + p_2 dY_2 = \mu_G p_2 dG$$

(81) $$\mu_G = \frac{dY_2}{dG} + \frac{p_1}{p_2} \frac{dY_1}{dG} = 1 + \frac{\alpha}{\beta} + k\delta p_1 \left(\frac{\gamma}{\beta} + (1 + \frac{\alpha}{\beta})\sigma_2\right)$$

. The employment multiplier λ_G is defined by :

$$dL_1 + dL_2 = \lambda_G p_2 dG$$

$$\lambda_G = \frac{1}{p_2} \frac{dL_1}{dG} + \frac{1}{p_2} \frac{dL_2}{dG}$$

(82) $$\lambda_G = \frac{1}{p_2 F_2'(L_2)} (1 + \frac{\alpha}{\beta}) + \frac{k\delta p_1}{p_1 F_1'(L_1)} \left(\frac{\gamma}{\beta} + (1 + \frac{\alpha}{\beta})\sigma_2\right)$$

./.

The aggregate Keynesian multiplier μ_G is the sum of those multipliers obtained in the two previous cases, where one of the sectors is in classical unemployment :

$\mu_G = 1 + \frac{\alpha}{\beta}$ sector 1 with classical unemployment
sector 2 with Keynesian unemployment

$\mu_G = k\delta p_1 \frac{\gamma}{\beta}$ sector 1 with Keynesian unemployment
sector 2 with classical unemployment ($\sigma_2 = 0$)

We also find that as sector 2 gets closer to a situation of classical unemployment (small σ_2), the aggregate multipliers become lower.

b) Effect of public investment I_G :

We obtain for sector 2 :

(83) $$dY_2 = F_2'(L_2)dL_2 = \frac{\alpha p_1}{\beta p_2} dI_G$$

and finally for sector 1 :

(84) $$dY_1 = F_1'(L_1)dL_1 = \delta \left[1 + kp_1(1 + \frac{\gamma}{\beta} + \frac{\alpha}{\beta}\sigma_2) \right] dI_G$$

The aggregate multipliers of public investment μ_I and λ_I for production and employment are defined respectively by :

$$\left| \begin{array}{l} p_1 \, dY_1 + p_2 \, dY_2 = \mu_I \, p_1 \, dI_G \\ dL_1 + dL_2 = \lambda_I \, p_1 \, dI_G \end{array} \right.$$

We obtain :

$$\mu_I = \frac{dY_1}{dI_G} + \frac{p_2}{p_1} \frac{dY_2}{dI_G}$$

(85) $$\mu_I = \frac{\alpha}{\beta} + \delta + kp_1\delta(1 + \frac{\gamma}{\beta} + \frac{\alpha}{\beta}\sigma_2) .$$

$$\lambda_I = \frac{1}{p_1F_1'(L_1)} \frac{dY_1}{dI_G} + \frac{1}{p_1F_2'(L_2)} \frac{dY_2}{dI_G}$$

(86) $$\lambda_I = \frac{\delta}{p_1F_1'(L_1)} (1 + kp_1(1 + \frac{\gamma}{\beta} + \frac{\alpha}{\beta}\sigma_2)) + \frac{\alpha}{\beta p_2F_2'(L_2)}$$

<u>REMARK</u> : By comparing the public consumption and investment multiplier μ_G and μ_I, we obtain :

(87) $$\mu_G - \mu_I = 1 + \frac{\alpha}{\beta} + k\delta p_1(\frac{\gamma}{\beta} + (1 + \frac{\alpha}{\beta})\sigma_2) - [\frac{\alpha}{\beta} + \delta + k\delta p_1(1 + \frac{\gamma}{\beta} + \frac{\alpha}{\beta}\sigma_2)]$$

$$= k\delta p_1 [\sigma_2 - \sigma_1]$$

From this we derive : $\mu_G \geqslant \mu_I \Leftrightarrow \sigma_2 \geqslant \sigma_1$ i.e. $\frac{p_2F_2' - w}{p_2F_2'} \geqslant \frac{p_1F_1' -}{p_1F_1'}$

and we see that $\sigma_2 \geqslant \sigma_1 \Leftrightarrow p_1F_1' \leqslant p_2F_2'$

Any public expenditure policy designed to increase the volume of production, should privilege the sector where the marginal productivity of labour is the highest.

Conversely, if we consider the employment multipliers λ_G and λ_I, we obtain :

(88) $$\lambda_G - \lambda_I = \delta [\frac{1}{p_2F_2'} - \frac{1}{p_1F_1'}]$$

$$\lambda_G \geqslant \lambda_I \Leftrightarrow p_1F_1' \geqslant p_2F_2'$$

Consequently, a policy aiming at an increase in employment should privilege the sector with the lowest productivity. The two objectives thus appear contradictory.

./.

c) Impact of the wage rate :

Other prices being constant, we have :

(89)
$$\begin{cases} F_2'(L_2)dL_2 = \frac{\alpha}{\beta p_2} \frac{\partial N}{\partial w} dw \\ F_1'(L_1)dL_1 = \frac{\partial A}{\partial q} dq + \frac{\partial A}{\partial w} dw \end{cases}$$

(90)
$$dq = \frac{1}{B_o + Q} [\frac{\gamma}{\beta} \frac{\partial P}{\partial w} dw + (p_2 F_2'(L_2) - w)dL_2 - wdL_1 - (L_1 + L_2) dw]$$

By using (90), we have :

(91)
$$dY_1 = F_1'(L_1)dL_1 = \delta [\frac{\partial A}{\partial w} - k(L_1 + L_2)] dw + k\delta [\frac{\gamma}{\beta} \frac{\partial P}{\partial w} + \frac{\alpha}{\beta} \frac{\partial N}{\partial w} \sigma_2] dw$$

According to the hypotheses, $\frac{\partial N}{\partial w}$ is positive but undoubtedly rather low ; $\frac{\partial A}{\partial w}$ is positive (capital-labour substitution effect).

An increase in the wage rate results in :

- an increase in activity in the consumer good sector induced by income expectations of households ;
- a twofold effect in the investment goods sector ;
  The first effect is the impact on investment which, as we have seen (§ 3.4), leads to a reduction of activity.
  The second effect, which is positive, is an indirect consequence of positive income expectations of households.

On the whole, the impact on activity and employment is therefore likely to be limited, though more favorable than in case 2, i.e. Keynesian equilibrium for sector 1, classical equilibrium for sector 2.

./.

d) Variation in price p_1 of capital goods :

(92)
$$\begin{cases} F_1'(L_1)dL_1 = \frac{\partial A}{\partial p_1}dp_1 + \frac{\partial A}{\partial q}dq \\ F_2'(L_2)dL_2 = \frac{\alpha}{\beta p_2}\left(\frac{\partial N}{\partial p_1}dp_1 + I_G dp_1\right) \\ dq = \frac{1}{B_o + Q}\left[\frac{\gamma}{\beta}\frac{\partial P}{\partial p_1}dp_1 + \left(\frac{\gamma}{\beta} + 1\right)I_G dp_1 + (p_2F_2'(L_2) - w)dL_2 - wdL_1\right] \end{cases}$$

By substituting expression dq, we finally obtain :

(93)
$$dY_1 = F_1'(L_1)dL_1 = \delta\frac{\partial A}{\partial p_1}dp_1 + k\delta\left[\frac{\gamma}{\beta}\frac{\partial P}{\partial p_1} + \frac{\alpha}{\beta}\frac{\partial N}{\partial p_1}\sigma_2 + \left(1 + \frac{\gamma}{\beta} + \frac{\alpha\sigma_2}{\beta}\right)I_G\right]dp_1$$

The increase in p_1 has a positive impact ($I_G dp_1$) and a negative impact ($\frac{\partial N}{\partial p_1} \simeq -\frac{\partial M_1}{\partial p_1}$) on the consumer goods sector.
It is difficult, without any further assumptions to reach a conclusion about the final result, as we have seen in § 3.3.

The effect on sector 1 is of a similar nature to the effect studies in § 3.4 ; $\frac{\partial A}{\partial p_1}$ is negative, but the second term, which represents a multiplier effect of public expenditure, is positive. Here too, the resulting effect is indeterminate.

e) Variation in the price p_2 of consumer goods :

The effects are defined by the following relations :

(94)
$$\begin{cases} F_1'(L_1)dL_1 = \frac{\partial A}{\partial p_2} + \frac{\partial A}{\partial q}dq & (1) \\ F_2'(L_2)dL_2 = \frac{\alpha}{\beta p_2}\left(\frac{\partial N}{\partial p_2} - \frac{1}{p_2}(N + p_1I_G)\right)dp_2 & (2) \\ dq = \frac{1}{B_o + Q}\left[\frac{\gamma}{\beta}\frac{\partial P}{\partial p_2}dp_2 + \frac{\gamma}{\beta}Gdp_2 + (p_2F_2'(L_2) - w)dL_2 + F_2'(L_2)dp_2 - wdL_1\right] & (3) \end{cases}$$

Substituting dq and taking into account expression (76) of $F_2(L_2)$, we finally obtain :

(95)
$$F_1'(L_1)dL_1 = \delta\frac{\partial A}{\partial p_2}dp_2 + \delta k\left[\frac{\alpha}{\beta}\frac{\partial P}{\partial p_2} + \left(\frac{\gamma}{\beta} + \left(\frac{\alpha}{\beta} + 1\right)\sigma_2\right) + \frac{\alpha}{\beta}\sigma_2\frac{\partial N}{\partial p_2} + (1 - \sigma_2)F_2(L_2)\right]dp_2$$

,

Unlike what we found when sector 2 was in classical equilibrium (§ 3.4), the increase in the price of consumer goods does not stimulate their production. Conversely, for the same reasons is mentioned earlier, activity rises in the investment goods sector.

In short, we have the following effects :

	G	I_G	w	p_1	p_2
Sector 1	+	+ +	?	?	+
Sector 2	+ +	+	+	?	-

CONCLUSION

To recap the three cases under study :

	G	I_G	w	p_1	p_2
Sector 1 : classical	0	0	- -	+ +	0
Sector 2 : Keynesian	+ +	+	+	?	-
Sector 1 : Keynesian	+	+ +	-	?	+
Sector 2 : classical	0	0	- -	0	+ +
Sector 1 : Keynesian	+	+ +	?	?	+
Sector 2 : Keynesian	+ +	+	+	?	-

In the three cases examined, increases in public expenditures have a stimulating effect : in each case, at least one of the two sectors is in Keynesian unemployment. The effect is direct when it involves the sector in Keynesian unemployment. Otherwise it is indirect. Given equal government expenditures, the direct effect is more substantial than the indirect one.

./.

The increase in the nominal wage rate results in the development of activity in the consumer goods sector when this sector is in Keynesian unemployment. Such an effect stems from the more favorable income expectations of households.

Conversely, the rise in nominal wages usually has a negative effect on the investment goods sector. The relative decrease in the cost of capital as compared to the cost of labour appears to be more than offset by the rise in the interest rate. Obviously, it always has a negative effect on a sector in classical unemployment.

The only clear cut effect of an increase in the price of investment goods is the stimulation of the corresponding sector when it is in classical unemployment. In the other cases, such an effect is not conclusive ; it is made up of two contradictory effects : a decrease in the demand for investment which results from the rise in cost and a reflation through an increase in the actual value of public demand. In the consumer goods sector, we again find the same effects, but expressed indirectly.

A rise in the consumer price has a negative effect on the corresponding sector when it is in Keynesian unemployment, and a positive effect when it is in classical unemployment. The impact on the investment goods sector is positive when this sector is in Keynesian unemployment.

The consequences of economic policies thus appear to be extremety different in effect depending on the sector to which they apply and on the nature of the equilibrium prevailing in the sector.

R E F E R E N C E S

[1] BARRO, R.J., and H.I. GROSSMAN : "General Disequilibrium Model of Income and Employment", American Economic Review, 61 (1971), 82-93.

[2] BARRO, R.J., and H.I. GROSSMAN : Money, Employment and Inflation. Cambridge : Cambridge University Press, 1976.

[3] BENASSY, J.P. : "Neo-keynesian Disequilibrium in a Monetary Economy", Review of Economic Studies, 42 (1975), 503-523.

[4] BENASSY, J.P. : "A Neokeynesian Model of Price and Quantity Determination in Disequilibrium", in Equilibrium and Disequilibrium in Economic Theory, ed. by G. SCHWODIAUER, DORDRECHT, 1977. (Proceedings of a conference held in Vienna, Austria, 1974), 511-544.

[5] CLOWER, R.W. : "The Keynesian Counterrevolution : A Theoretical Appraisal", in The Theory of Interest Rates, ed. by F.H. HAHN and BRECHLING. London : Mcmillan, 1965.

[6] DREZE, J. : "Existence of an Equilibrium under Price Rigidity and Quantity Rationing", International Economic Review, 16 (1975), 301-320.

[7] GRANDMONT, J.M. : "Temporary General Equilibrium Theory", Econometrica, 45 (1977), 535-572.

[8] GRANDMONT, J.M., and G. LAROQUE : "On Keynesian Temporary Equilibria", Review Economic Studies, 43 (1976), 53-67.

[9] HILDENBRAND, K. and W. HILDENBRAND : "On Keynesian Equilibria with Unemployment and Quantity Rationing", Journal of Economic Theory, 18 (1978), 255-277.

[10] HOOL, B. : "Monetary and Fiscal Policies in Equilibria with Rationing", SSRI, University of Wisconsin, Madison, 1977.

[11] MALINVAUD, E. : The Theory of Unemployment reconsidered. Oxford : Basil Blackwell, 1977.

./.

[12] MALINVAUD, E., and Y. YOUNES : "Une Nouvelle Formulation Générale pour l'Etude des Fondements Microéconomiques de la Macroéconomie", Cahiers du Séminaire d'Econométrie du C.N.R.S., 18 (1977), 63-112.

[13] MUELLBAUER, J., and R. PORTES : "Macroeconomic Models with Quantity Rationing", The Economic Journal, 88 (1978), 788-821.

[14] YOUNES, Y. : "On the Role of Money in the Process of Exchange and the Existence of a Non-Walrasian Equilibrium", Review of Economic Studies, 42 (1975), 489-501.