N° 7902 BIS

THE "SO-CALLED TRANSFORMATION PROBLEM" REVISITED

by

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A lot of ink has flown since Von Bortkiewicz criticized the way Marx dealt with the transformation problem between values and prices of production in Volume III of Capital [11]. Of course, today, nobody would claim this "mistake" did open "the crisis of marxism" (1). Such a petty problem deserves neither so much glory nor such injury. Yet, today the commonly admitted rectification (let us take for reference that of M. Morishima (2)) implies a considerable lost of prestige of the marxist theory of value. That one is brought back to a rather primitive and approximative formulation of the notion of labor exploitation. At least, it is the interpretation of M. Morishima himself (with his "fundamental marxian theorem" : "the equilibrium rate of profit is positive if and only if the rate of exploitation is positive"), as well as that of P.A. Samuelson. For the latter,

> *"Although Capital's total findings need not have been developed in dependence upon Volume I's digression into surplus values, its essential insight does depend crucially on comparison of the subsistence goods needed to produce and reproduce labor with what the undiluted labor theory of value calculates to be the amount of goods producible for all classes in view of the embodied labor requirements of the goods. The tools of bourgeois analysis could have been used to discover and expound this notion of exploitation if only those economists had been motivated to use the tools for this purpose."* (3)

"The tool of bourgeois analysis" : that is to say, the theories according to which the time of the owners' saving, being as scarce a good as labor, also deserves a specific income (the interest).

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In such a situation, many marxist authors, especially in France, prefer to stick to a prudent denial of Morishima-type of formalisations, objecting criticisms rather irrelevent to the point in discusion. Some of them even refuse, because of too strict epistemologic criteria, to deal with the transformation problem itself (4).

The chief aim of this paper is to show that the Morishima-type of solution, reevaluated and completed, is not contradictory to the main claims of Marx's Capital, but still there is another solution (5) more faithful to the approach of Capital, and which exhibits the famous equalities of Volume III discarded in Morishima's solution : "Sum of prices = sum of values ; Sum of profits = sum of surplus values". Moreover, it will be shown that for any given output structure, the rate of profit is dependent on this structure and not on the workers' consumption structure (as it is implied by M. Morishima).

The first part will be concerned with Marx's solution and its criticisms, the second one with the Morishima type of solution, the third one with the new solution, and the last one with a comparison of these two solutions.

I - MARX'S APPROXIMATIVE SOLUTION.

For Marx (6), the commodity exchange character of the economy confers a "value" to the output of economic units. This value is proportional to the part of social abstract labor allocated by society to their production and socially validated in exchange (that is to say : recognized as useful by being purchased). In a modern, algebric formalisation, each bundle of commodities can be represented by a vector y in a space naturally spanned by the n different units of use-value. Thus, value is a linear form ν on this space, which maps y into a positive real number (7).

In a supposed standard productive operation, the "living labour" applied to means of production adds value to the value already embodied by "past labour" in the means of production (8). So, let a_j^i be the quantity of good i which is necessary to the standard production of good j, A be the "technical matrix", $\ell = [l_1 \ldots, l_j]$ the quantity of value embodied by abstract labour when applied to the production of one unit of good 1,...j,... The value covector is thus defined by :

$$\nu = \nu A + \ell$$

Or :

$$\nu = \ell (I-A)^{-1}$$

Let us remark that this way of "adding" living labour to embodied value (9) arises some reserves, and is refused by some French marxists (10). In fact, it is quite admissible, as far as the norms of production remain constant, and as far as economy is studied in its reproduction. Then, the "embodied value" of the inputs of some sector is equivalent to the present labour applied in some other sector. Of course this does not hold when an evolution of productive forces is assumed, and it is part of a marxist explanation for inflation and crisis (11). Anyway, in all the "transformation controversies" the two assumptions hold.

Up to this point, nothing has been said about the capitalist character of our economy. The marxist approach, as any scientific approach, goes on by successive approaches of the abstract determinations of concrete reality (12) : first one studies the fall of bodies in the void, then one introduces air resistance, and so on. In the first section of volume I, Marx lays down and studies the law of value (substance, measure, form of value) for any commodity production. Then he studies the partitions introduced by the capitalist relations in the measure of value (C/V/PL). Later (in Volume III) he introduces the transformations in the forms of the law of value itself, induced by the capitalist relations.

With capitalism, the labour power appears to the capitalist as a commodity, with a value w. This value is the amount of labour time, which the workers got the right to spend on the market when purchasing consumption goods in order to reproduce their labour power daily. This commodity has a use-value elsewhere : to product abstract labour, and thus to add value. The amount of abstract value v_a that can be extracted from the commodity "labour power" is defined by the duration λ and the intensity ε of labour. The three data w, ε, λ, are the results of a historical process, of the "class struggle" (13). Together, they determine the rate of surplus-value e, which is the ratio of the "unpaid labour" $v_a - w$ to the value of the labour power w. Thus, by definition :

$$w\ (1+e) = 1$$

In the vector formalisation, we have now :

$$\nu = \nu\ A + w\ell + ew\ \ell$$

That is the modern form for Marx's C+V+PL. It is to be noted that, in these formalisations, we assume that the quantity of the commodity "labour power", which capitalists need to purchase and set to work in order to produce the good j, is measured by the same number as the quantity of abstract labor. This implies that we have taken the intensity and length of the working day for data, and that we have chosen one "worker-day" (for instance) as unit of labour power, and the abstract labour produced in one day as unit of value. And, of course, the productive operations being assumed standard, concrete labor is identified with socially-necessary labour. In other words, the existence of a tensor T is to be assumed, mapping n-uples of commodity "labour power" into covectors of "value added", and the coefficients of which are defined by intensity and duration of labor (14). These are petty trivial things, but, in forgetting them, we are induced to identify "commandable labor" and "embodied labour", just as the classical pre-marxist authors Smith and Ricardo did (15), and (that is more important in our discussion) to identify a <u>social</u> relation and a <u>technical</u> relation between "quantities of input and output".

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The illusion of a pure technical relation between input and output is complete when it is assumed that, just as the production of a good j is summarized by the coefficients of the standard productive operation (a^i_j, l_j), so there exists a standard consumption bundle corresponding to one worker-day : d. Then :

$$w = \nu . d$$

and the existence of the quantity l_j of labour-power fades away into the data of a bundle of commodities indirectly required by the production of j : the vector $[d^i \, l_j]$, added to the vector $[a^i_j]$. Now we have a "technico-social" matrix :

$$M = A + d \otimes \ell \quad (= A + d \otimes \quad (\ell \, T^{-1}))$$

($\otimes$ is the sign of the tensorial product, or, in the matricial representation, of the Kronecker product).

This matrix looks purely "technical", but the three determining elements of the theory of value and exploitation are <u>already</u> incorporated in it :

- by the ν - measure of d (as far as value of labour-power w is concerned)
- by T, the (maybe implicit) tensor mapping quantity of labour-power into quantity of embodied labour (as far as λ and ε are concerned)

But the transformation of the law of value is only beginning. If exchanges in a pure commodity economy are regulated by the value of products through competition, how are exchanges between economic units regulated when these units are individual capitals, when "commodities are products of capital" (16), that is to say, of labour engaged by capital, not of labour alone ? In Marx's view, the answer must be : a transformed value, so that the quantity of surplus-value (called then "profit") returning to each capitalist be in proportion to engaged capital. The value produced in one period by the Society "working as a unique force", is then realloted onto outputs (by competition between individual capitals (17)), the "transformed value" or "price of production" being settled by the condition upon the equalization of profit rates. So, the sum of values will be equal to the sum of prices of production, and the sum of surplus values will be equal to the sum of profits, or, at least, the ratio of these two pairs of quantities (a ratio depending on the choice of the numéraire) must be the same.

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Is there a mathematical possibility for that ? Yes, is Marx's answer, for it is the mere consequence of the theory of value and exploitation. And he presents a simple algorithm, in Volume III (unachieved at his death).

Let us assume a partition of economy into industries or sectors i, each of them producing a value : $M_i = C_i + V_i + PL_i$, (in our formulation : $M_i = v_i\, y^i$, y^i being the physical output of i). In each sector the value of engaged capital is $C_i + V_i$. Total surplus-value is PL_i. The general rate of profit is :

$$r = \frac{\Sigma\ PL_i}{\Sigma\ (C_i+V_i)}$$

If each sector is to realize the same rate of profit, then it is sufficient to "equalize" only the surplus-value, and the output is thus valued in production-prices :

$$PP_i = (C_i + V_i)\ (1+r)$$

Of course, then, the two "invariance conditions" hold. And the average rate of profit is determined :

- by the rate of surplus value e
- by the organic composition of capital of various sectors C_i/V_i
- by the relative weight of the variable capital allocated to the different sectors (hence by the vector of outputs y).

This model suffers of two limitations. Firstly, all sectors are assumed to have the same turnover period. Marx developped many calculus in order to evaluate the consequences of this simplification. But that is not the clue of the controversy. Simplifications are natural within a scientific approach, and we shall stick to this one in the text (18). No, the criticisms hold upon another simplicication, this one less justifiable : capitalists do not purchase conditions of production "at their value" C_i and V_i , but at their price of production. The "reallocation" of value operates not only upon PL_i but also upon C_i and V_i. That point is clearly noted by Marx himself (19), but he thinks that the point is of weak concern, and he goes on to what he thinks to be more important. What a mistake ! People don't forgive easyly this kind of benign neglect from great thinkers... And

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unfortunately, authors who have taken upon the correction of this neglect made it in such a way that they cancelled the main idea of Marx. Namely : the profit, far from being the "reward of abstinence", is *actually* an unpaid part of the value created by productive labour.

II - THE ACCEPTED SOLUTION TO THE TRANSFORMATION PROBLEM.

In order to cope with Marx's neglect, we must assume that labour and means of production are now purchased "at their price of production". What does that mean ? As far as C_i is concerned, there is no problem : it means the valuation of the commodities "means of production" by the prices of production. But what does it mean "to purchase labour-power at its price of production" ? Actualy, V_i is but an amount of money, granted to workers, and representing a fraction of their value added (this fraction being defined by the rate of surplus-value). Yet the first "transformers" stroke upon an expedient that will turn out not to be neutral. Their solution consists in valuating the labour-power, not directly by this share of value added, but indirectly by the value of the commodity-bundle d that this part would purchase, when it is assumed that all workers purchase the same bundle and spend their money on a market where relative prices are regulated by the system of values.

Let us admit it, for a while. Then, V_i is transformed just alike C_i, by valuating d according to the same prices of production. Let be p the covector of prices and r the average rate of profit.

$$\text{Now :} \quad p = [p\,A + (p.d)\,\ell]\,(1+r)$$

$$\text{Or :} \quad \frac{1}{1+r}\,p = p\,[A + d \otimes \ell] = p\,M$$

Thus p is the left-eigenvector corresponding to the eigenvalue $\frac{1}{1+r}$ of the technico-social matrix M. But p is non-negative, and so is M. The Perron-Frobenius theorem (20) tells us that p must be eigenvector associated with the root of Frobenius μ (M).

$$\text{Thus : } r = \frac{1}{\mu\ (M)} - 1$$

First remark : μ depends only on M, that is on A, p and d (which elsewhere define e). Among the set of vectors d such that $v.d = w$ (that is : for a constant rate of surplus-value), r depends only on the direction of d (that is : on the working class consumption structure). *And not at all on the structure y of total output.*

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Yet for the marxist tradition there is something still worse. Let us take the *numeraire* such as : sum of prices = sum of values, or $p.y = v.y$. Then :

Sum of profits = $r\ p\ M\ y$

Sum of surplus-values= $e\ w\ \ell.y$

Theses sums are not equals but iff :

$$(r\ p\ M - e\ w\ \ell).\ y = 0$$

That means : iff y belongs to a n-1 - multiplicity. Yet a priori y belongs to a n-multiplicity (21). Thus, excepted on a zero-measured subset, the "invariarance condition" does not hold :

$$\frac{\text{Sum of prices}}{\text{Sum of values}} \neq \frac{\text{Sum of profits}}{\text{Sum of surplus-value}}$$

Faced with these two results, many marxists prefered to veil their face. They are wrong, in my opinion. Not only because M. Morishima, Okishio and Seton supplied this little drop of comfort, the "fundamental marxian theorem : r > o iff e > o" (22). But also because this conceptual framework provides nearly all the learnings Marx was awaiting from his model.

1°) The value of commodities recovered by capitalists is in fact the surplus-value.

In other words : even if the sum of profits differs from the sum of surplus-values, the value of the uses of profits is actually the social surplus-value. The proof of this theorem is quite trivial. It is simply to be noted that, when writing :

$$p = (1+r)\ p\ M$$

we mean that all production is realized, there is no over-production. On the over hand, the product of the period, y, is needed :

- for the reproduction of the conditions of production : $M\ y$
- for the improductive consumption of capitalists : C
- for the production of the conditions of accumulation : $M\ \Delta y$

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The uses of profits are represented by the last two terms.

Thus :
$$y = My + C + M\,\Delta y$$
$$v.\,(y - My) = v\,(C + M\,\Delta y)$$
and
$$e\,w\,\ell y = v\,(C + M\,\Delta y)$$
Q.E.D.

In a more intuitive way, it is clear that if capitalists are to sell commodities at prices diverging from values, they are to purchase commodities from each other in the same conditions, and there must be some compensation, in a way we are now to explain.

2°) Prices of production regulate the behaviour of "capitalist adequate to his concept".

In the last paragraph, the "improductive consumption" C of capitalists was undetermined, and that was inherent to the level of abstraction wich is ours. C could not be determined but by new psycho-sociological considerations. In theory, the capitalist, according to Marx's words, is but the "functionary of its own capital", that is, of a value the only claim of which is its own valorisation. So let us reduce capitalist to his essential character:he would obey the famous calvinist ethic according to Max Weber. Exceedingly frugal, he would accumulate all his profits, and into his own sector, for he has no reason (provided there is no other determination) to behave another way.

So let $p_i\,y^i(t)$ be the turnover of the sector i at time t. It is completely dedicated to the purchase of the conditions of expanded production in the next period $y^i\,(t+1)$:
$$p_i\,y^i\,(t) = (p\,M)_i\,y^i\,(t+1)$$
$$= \frac{1}{1+r}\,p_i\,y^i\,(t+1).$$

Thus : $$y^i\,(t+1) = (1+r)\,y^i\,(t)$$

Or : $$y\,(t+1) = (1+r)\,y(t)$$

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On the other hand, these conditions of production (including consumption goods for newly recruited workers) constitute the whole gross output of period t :

$$y\,(t) = M\,y\,(t+1)$$

so : $$y\,(t+1) = (1+r)\,M\,y\,(t+1)$$

The gross output vector is therefore $\hat{y}$, the right eigenvector associated with the Frobenius root $\mu(M)$; its growth goes on at the rate r : it is the famous "balanced growth path" that M. Morishima calls "Marx-Von Neumann model" and which makes steady the maximal growth (23). Let us call it "integral accumulation model".

For this structure $\hat{y}$ of production (of zero-measure in the set of all feasable structures, but enjoying an undisputed legitimacy !) it is easy to prove that, when chosen the production prices at a level $\hat{p}$ such that $\hat{p}\,.\,\hat{y} = \nu\,.\,\hat{y}$, then :

$$\text{sum of profits} = r\,\hat{p}\;M\,\hat{y} = \frac{r}{1+r}\,\hat{p}.\hat{y} = \frac{r}{1+r}\,\nu.\hat{y}$$

$$\text{sum of surplus values} = \nu.\hat{y} - \nu\,M\,\hat{y} = \frac{r}{1+r}\;\nu.\hat{y}$$

So : *sum of profits = sum of surplus values.*

These two systems of reference $\hat{y}$ and $\hat{p}$ (24) now allow us to give a more accurate notion of the "compensation" (refered to in the last paragraph) of the divergence between values and prices by the structure of production, as soon as the problems of realisation and accumulation are taken into account. As a matter of fact, any production vector y may be split in a unique way into one component $\hat{y}$ on the integral accumulation structure and one component $\tilde{y}$ in the hyperplane orthogonal to $\hat{p}$, so that :

$$\begin{cases} y = \hat{y} \oplus \tilde{y} \\ \hat{p}.\hat{y} = \nu.\hat{y} \\ \hat{p}.\tilde{y} = 0 \end{cases}$$

Let us denote $\delta\,\nu = \hat{p} - \nu$ the "divergence" from ν to $\hat{p}$

$\delta\,y = y - \hat{y}$ the "divergence" from y to $\hat{y}$

So : $$\delta\,\nu\,.\,y = (\hat{p} - \nu).y = \hat{p}.\hat{y} + \hat{p}.\tilde{y} - \nu.\hat{y} - \nu.\tilde{y} = -\,\nu.\tilde{y}$$
$$= -\,\nu.(y-\hat{y})$$

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Thus : $\boxed{\delta v \, . \, y + v \, . \delta \, y = 0}$

In other words, we can propose the :

<u>Theorem of price-value compensation</u>. When the structure of the gross output is deviated from the integral accumulation model, the v-valuation of this deviation is equal to the valuation of the divergence between value and prices of production by this output (understood as a linear form on the set of covectors).

So there is a connection between the structure of output and the divergence from value to prices. But it looks as a much weaker connection than in Marx. Up to here, $\hat{p}$ and $\hat{y}$ are both functions of M, hence of d, and y is but a mean to estimate ex post their correlative variations.

3°) <u>The rate of profit is a well defined function of the rate of surplus-value</u>.

If one is unsatisfied with the "marxian fundamental theorem", one may try to get explicitly the one-to-one function that links e and r through the parametric data A, d, ℓ. But, as far as d is concerned, the form : $r = f_d\,(e)$ is unsatisfying, because d is itself constrained by e (namely : $(1+e)\; v.d = 1$). So we must seperate more clearly the variable (the intensity of exploitation) and the other social parameters (the orientation of workers' consumption). How can it be done ? I will indicate two ways.

The method of G. Dumenil [7] consists in decomposing the bundle d of consumption into its v-norm w, and the data of the structure of consumption d^* :

$$\begin{cases} d = w\, d^* \\ v.d^* = 1 \end{cases}$$

So : the working class is exploited at a rate e (recall : $w(e+1) = 1$) and "chooses" a consumption structure out of the simplex $v.d^* = 1$.
G. Dumenil and C. Roy have computed the mapping (25) :

$$r = f_{d^*}\,(e)$$

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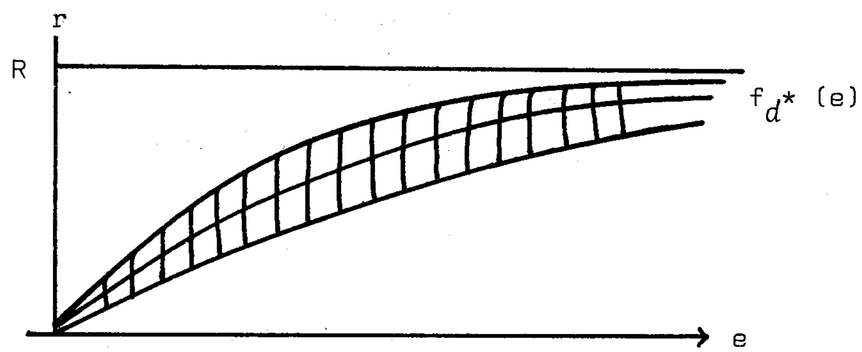
The d^* - indexed curves are convex and strictly increasing with the range from 0 to R, as e runs from 0 to infitity (R being the root of Frobenius of the "technical matrix" A : that is the limit of M when the workers "live out of thin air"). The enveloppe of this family of curves is a set of arcs of peculiar curves : namely, the curves corresponding to peculiar d^* that minimizesand maximises the organic composition evaluated in a specific way.



Another method, more sophisticated, is supplied by J. Roemer [13]. Here, each worker is allowed to choose his bundle of consumption goods, given a social rate of surplus value. Each worker is indeed alloted a choice function γ (accepting neo-classical properties of continuity and convexity). Wage and prices being given in the prices of production system, each one thus "freely" chooses how to spend his income. By combining the theorems of Perron-Frobenius and Brouwer, J. Roemer shows that there exists only one system of relative prices so that workers choose bundles of consumption respecting the value e of the rate of surplus value (26). The correspondance between e and r is now indexed by Γ (the set of all workers choice functions), but the shape of the family of curves $r = f_\Gamma$ (e) is the same as in Dumenil.

Yet, the use of Brouwer's Fixed Point Theorem (which gives the result of an algorithm without any economic meaning) sets a fundamental problem. How is it possible for consummers to choose a bundle of goods out of a price-regulated market when their budget constraint is given in the value system (27) ? This problem refers to the highly questionable expedient which is common to all the Morishima-type of solutions to the transformation problem. Namely : the one "transforming V" out of the value of consumption goods. Once again, let us recall that the workers are paid off with money, and not with a right on a commodity bundle. When the "transformation" is realised, when the choices are expressed in the prices system, and if all the workers choices are alike, the relation $r = f_{d^*}(e)$ is surely true. But it is not obvious that it would correspond to a causal scheme : $(d^*, e) \rightarrow r$. Anyway, in Marx's view, the causal scheme is $(y, e) \rightarrow r$. Up to here, we stick to the same weakness of the admitted solution to the transformation problem.

We shall come back to this point later.

4°) The prices of production and the rate of profit are determined out of the theory of labour-value and exploitation.

Here is the clue of the controversies. So strong as could be the connections (that we have just studied) between the system of values and surplus-values on one hand, and the system of prices and profits on the other hand, these connections, at the first glance, look like "cousinhood bounds" and not "filiation bounds" (the common ancestor being the "technical" data of the coefficientsof $A + d \otimes \ell$).

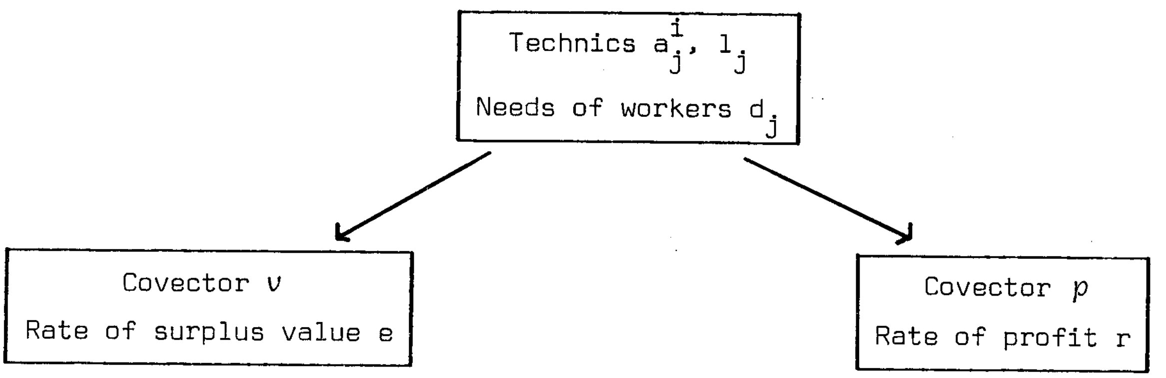


Figure I

As a matter of fact, the matrix M appears to determine a "surplus" (under the "technical" condition of its productiveness), and this net product is to be alloted according to two systems :

- either on the prorata of living labour added during the last period
- either on the prorata of the cost-price.

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P.A. Samuelson [15] compares these two systems to the incidence on prices formation of a value added tax on one hand, of a turnover tax on the other hand. In the first case, the resolution of the system of equations is simple enough : it is a system of n linear equations. In the second case, an algebric equation of degree n is to be solved (28) !

> "*One might apply,* P.A. Samuelson suggests, *Marx's theory of the materialist determination of history to arrive at the hypothesis that it was Marx's incapacity in algebra and the absence of a computer that caused him to formulate his exploitation theory in Volume I terms which are unrealistic but which happen to be simpler to handle algebraically than Volume III's Walrasian relations*". (29)

Henceforth the "so-called transformation problem" is quite easyly solved (on the basis of A, d, ℓ being given)

> "*The "transformation algorithm" is precisely of the following form : "Contemplate two alternative and discordant systems. Write down one. Now transform by taking an eraser and rubbing it out. Then fill in the other one. Voila ! You have completed your transformation algorithm*""(30).

Unfortunately, this happy end of the affair implies that matrix M (out of which, true, p and r are directly deduced) would be logically given *before* the theory of value and exploitation (31). But we have already seen (part I) it was not the case. Let us come back to the logical chain, link after link.

At first, we have the commodity-exchange character of economy. Given the state of productive forces (A, ℓ) one computes the value-covector ν. Up to here : not a word about exploitation, nor surplus-value, nor profit, nor production prices, nor wages, nor M (32).

Let us introduce the wage-worker-to-capitalist relation, and the purchase and use of the commodity "labour power", defined by its value w, and the duration λ and intensity ε of the working day. These 3 factors co-determine surplus value, hence e. The value of labor power corresponds, through ν, which is already computed, to a purchasable bundle of necessary consumption goods. At least, as far as the Morishima-type (and Samuelson-type) of solution for the transformation of w is accepted.

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All these terms : λ, ε, w, e, d, are directly or indirectly, the goal of struggle relations between social classes, and are connected in a complex way.

* λ and ε are slightly connected, and, much more weakly, connected to d (33). We are allowed to take them for data before the computation of the rate of surplus-value. Anyway, they are implied in the definition of tensor T mapping n-uples of commodity "labour power" into covector "value added". This mapping (which is quantitatively speaking an identity when the units are *properly* chosen) is the obligatory link from the "technical" data (A, ℓ) to the equations of production prices :

$$p = (1+r)\ (p\,A + \omega\,\ell) \quad (\omega = \text{rate of wage})$$

This form of the system is indeed an abreviation for :

$$p = (1+r)\ (p\,A + \omega\,\ell\,T^{-1})$$

* λ and ε being given, the connections between e, d and w are far more complex. The *dynamic* process that appears to be directing is of this type :

$$e \rightarrow w \rightarrow d$$

In other words : capitalists and workers face each others ; the bargaining (with or without strikes, riots and so on) settles the ratio e, hence the value w, and workers spend their money on the market.

Yet in the Chapter VI, Volume I, of *Capital*, Marx, in order to give a more obvious meaning for the statement "the labour power, as any commodity, is given a value, namely, the quantity of labour necessary to its reproduction", gives a little strain to the analogy in laying the existence of a standard workers consumption vector, that appears like a kind of input of the firms "households". Let us notice, *en passant*, that the "operative" of this kind of firm (the wife, direct producer) works for no wage, and the "manager" (the husband) offers the output at its cost-price, with no mark-up on the input. Leontieff, Von Neumann and Morishima just codify this representation of the worker as a draught-animal claiming for its peck.

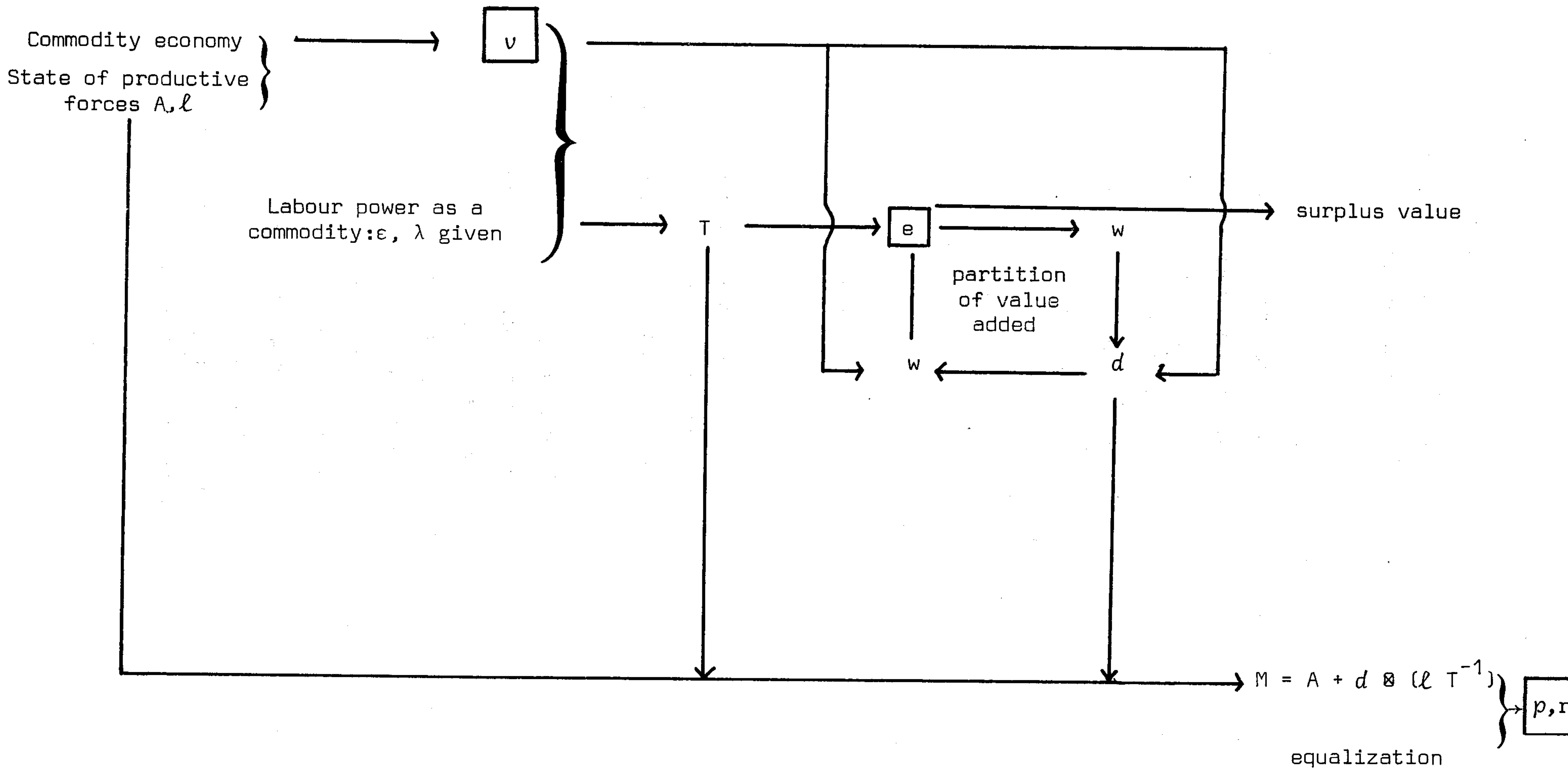
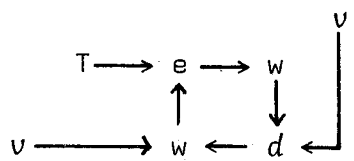

Commodity economy
State of productive forces A,ℓ
v
Labour power as a commodity:ε, λ given
T
e
w
surplus value
partition of value added
w
d
M = A + d ⊗ (ℓ T⁻¹)
p,r
equalization

Figure II

In fact, Marx, in the Capital, quickly drops out that physical determination of a "wage-bundle", and turns out to the determination of wage as a quantity of paid labor (34). But the way of recognizing the existence of a "standard consumption-bundle", for one time, in one country, is not an absurdity. Of course, trade-unions would not directly negociate for the right to a dish-washer or a colour - TV - set : once again, the bargaining applies on wage-rate. Yet, the physical standard of living, once being settled, cannot be lowered without difficulty. Not for moral considerations, but... because it would induce the breakdown of corresponding industries (35) !

So, in a marxist terminology, we could say that e and d are "dialectically connected", d being the "basis" and e being the "directing factor". Hence, the logical chain looks like this :



Now, d being defined, and wage being used to purchase the consumption goods "at their prices of production", we may conclude with the last link of the logical chain, represented in figure II. The comparison with figure I justifies the somehow cautious way we have decorticated the signification of algebric symbolism in the first part of this paper. Even if we accept d as an exogenous data, the theory of value and exploitation turns out to be presupposed to the theory of prices of production (at least : through tensor T).

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III - A NEW SOLUTION TO THE TRANSFORMATION PROBLEM.

The today commonly admitted solution has turned out to be less paradoxal for an "orthodox marxist" view than usually thought. Yet, it still diverges from Marx's claims.

* If the theory of value turned out to be actualy a pre-requesit for the formalism of computation of prices of production, it is not clear in the algorithm of transformation. One would like "to see" the value being realloted on outputs according to the mecanism of "equalization". That was the real aim of the two "invariance relations" (Sum of values = ... and so on).

* If there does exist a connection between the structure of production and the deviation from value to production-prices, it appears "ex-post". In Marx's view, the structure of output y (not of workers' consumption d^*), for a given rate of surplus-value, is to determine the rate of profit through the ponderation of organic compositions of capital in the various sectors.

1°) The crux of the matter (36).

In fact, all these divergences between Marx's intuitions and the Morishima-type of solution have their origin in the very way the "transformers" from Von Bortkiewicz resolved the peculiar problem of the *transformation of the value of labour-power*. Their common solution consists in transforming "V" (or here : w) just alike "C". Nobody would deny that, given a technology (A, ℓ) (37), the cost price of constant capital C_j is $\sum_i p_i a^i_j$ (per unit). But is it allowed to reduce the labour power to the output corresponding to an input d and a cost-price $\sum p_i d^i$? That would be at most an approximation for the economics of a "quasi-esclavagist mode of production", that is to say : esclavagist economy *within* the units of production, and commodity-economy *between* the units of production (for instance : in Dixieland before the Civil War) (38). On the contrary, the workers recover from capital an amount of *money* and choose what they would purchase according to their needs (denoted Γ in J. Roemer). In fact, the "way of life" constraint compels them to purchase a "fuzzy bundle" of consumption goods, and the value of this bundle for a peculiar state of

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"class strugle" provides a basis to the value of labour power. But it does not follow that one may extract the univoque determination :

$$\begin{array}{ccccc} & & d & & \\ & \swarrow & & \searrow & \\ p \rightarrow & s & & w & \leftarrow v \\ & \downarrow & & \downarrow & \\ & r & & e & \end{array}$$

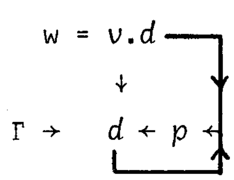
We are rather compelled to formalize this kind of linkage :

$$\begin{array}{ccccccc} & & e & \rightarrow & w & & \\ & & \uparrow & & \downarrow & & \\ v & \rightarrow & w & \leftarrow & d & \leftarrow & v \end{array}$$

with a "loop" $d \longleftrightarrow e$. But the "mediation" :

$$\begin{array}{ccc} w & & \\ \downarrow & & \\ d & \leftarrow & v \end{array}$$

is neither admissible, for the workers are to spend their wages "in the production-price system". The quoted paper from J. Roemer is a better approximation to a solution. His using of Brouwer Fixed Point Theorem is a formalization for this kind of pattern :



But no economic reality corresponds to the mathematical algorithm : the workers should purchase goods "at their prices of production" within a budget constraint ($v.d = w$) defined... in the value system !

The reason for it is that, in J. Roemer, the value of labour power, instead of being defined as the paid share of their added value, ($w = \frac{1}{1+e}$), sticks in fact to the bare value of a commodity bundle, even if the bundle is no more defined ex ante (but it remains presupposed to the mapping $\mathcal{D} \rightarrow \mathcal{D}''$ upon which the Fixed Point Theorem is applied).

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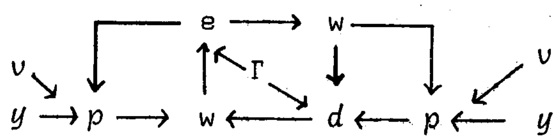
On the contrary, everything is made clear when w is considered as a "quantity of paid labour", a share $\frac{1}{1+e}$ of the added value, to be spent according to socially defined needs Γ (39), on a market regulated by prices of production, that prices being defined by a reallocation of value, and that reallocation *not* depending on d, but, like in Marx, on y. The result of these outlays turns out to be a bundle d (or a familly of vectors d for various workers), the value of which may well be different from w, in terms of embodied labor, not in terms of realloted value. This bundle, if unsufficient according to Γ, may henceforth be considered as a basis for a new bargaining of the partition of value added. The logical scheme is then :



This solution, which implies the necessity of dealing in a different way with constant capital C and variable capital V, happens to be conform with Marx's indications upon the way of achieving the transformation properly. Of course, it is not a proof ! But when intending to formalize Marx's thought about the transformation from value to prices of production, one is supposed to check up wether it is the very thought of Marx that is formalized, or the thought of somebody else. And what is Marx's message, after he has himself criticized his clumsy little model of transformation ? He writes that now the elements of cost-price are to be transformed this way :

> "*In addition to the fact that the price of the product of capital B for instance diverges from its value because the surplus-value embodied in B can be superior or inferior to the profit within the price of product B, the same circonstance occurs also in commodities which constitute both the constant fraction of capital B and, indirectly, as means of subsistence of workers, its variable fraction. As far as the constant fraction is concerned, it is itself equal to the cost of production plus the mean profit (...). As for as the variable fraction is concerned, the mean daily wage is still in fact equal to the value produced during the nomber of hours the worker works in order to produce its necessary consumption goods. But the divergence between the value and the production prices of these commodities falsifies that number of hours itself*". ([11], Volume III, chapter X. Translation from the french edition, to be substituted when the Penguin edition available in Paris !).

In other words, in Marx's thought, as far as constant capital is concerned, we just have to transform the valuation of a physical input. But as far as variable capital is concerned, wage, considered as a share of value added, as a "number of hours", is conserved within the transformation, but the number of hours itself, considered as equivalent to a peculiar bundle of necessary commodities, is transformed ! That is exactly our last scheme.

But we are now to check up the mathematical consistency of the whole story.

2°) <u>The formalisation</u>.

Let ν be the value of commodities (ν is computed from A and ℓ). Let e be the given rate of surplus value, hence w the value of labour power. We are looking for a reallocation of total value embodied within the period, that is : reallocation of the total flow of abstract labour onto the total output corresponding to this labour, in other words the net output of the period. Henceforth we shall denote y this net output, and Y the gross output ($y = Y - AY$).

This transformation must be really a *reallocation*, thas is a reallocation of the same quantity of labour value. Let $\hat{p}_i$ the realloted value on the unit of good i. The covector $\hat{p}$ defines the system of relative prices (the nominal prices depend on the choice of the *numéraire*). By definition of a "reallocation", if such a system exists, it must verify :

$$(H_1) \qquad \hat{p}.y = \nu.y$$

On the other hand, this reallocation must realize a *capitalist equalization* of surplus value. In other words, the realloted value p_i must be equal to γ times the sum of constant capital (evaluated in realloted values) and of variable capital (evaluated as the value granted to workers in exchange for the disposition of their labour-power) engaged in production of one unit of i, and γ must be equal in any sector. γ is nothing but 1+r,

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so one expects $\gamma > 1$. Assume as usual that the unit of value and the unit of labour power are such that the tensor T may remain implicit. Then, by definition of capitalist equalization, if the system $\hat{p}$ exists, it must verify :

$$(H_2) \qquad \hat{p} = \gamma\,(\hat{p}A + w\,\ell)$$

This condition can be expressed in another way. We want the "commended labour" (40), direct (during the last period, indexed as 0) or indirect (during the former periods indexed n), to contribute in the reallocated value $\hat{p}_i$ of the output i according to the proportion γ^{n+1} (41) :

$$(H_2') \qquad \hat{p} = w \sum_0^\infty \gamma^{n+1} \ell A^n$$

(H_2') is equivalent to (H_2) which may indeed be written this way :

$$(H_2'') \qquad \hat{p} = w\,\ell\,[\frac{I}{\gamma} - A]^{-1}$$

and that formulation can be expressed as (H_2') by Taylor's expansion of the inverse matrix (42).

Notice that the same operation can be applied to :

$$\nu = (1+e)\ w\ \ell\ (I-A)^{-1}$$

$$\text{Thus} \qquad \nu = (1+e)\ w\ (\sum_0^\infty \ell\ A^n)$$

Now we understand clearly that the reallocation consists in a redistribution of *the whole value* (43) so that the total embodied surplus-value is alloted, not on the prorata of commended labor during each past period, but on the prorata of all the past labours weighted by γ^{n+1}. P.A. Samuelson's comparison with value-added-tax and turnover-tax is good but insufficient, for the "turnover" is useless. Let us assume that a spinning factory buy a weaving factory. The spinners' labour, which was discounted as constant capital for the weaving firm, is now discounted as variable capital... but is tied up during two periods. That fact is correctly taken into account in formulation (H_2').

The system of realloted value which we are looking for (and out of wich any system of prices of production can by deduced by the choice of a numéraire) is thus well defined by (H_1) and (H_2). Now the question (Marx's question) is : does such a system exist, and what are its properties ? It

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will be quite easy to prove the following theorem, summing up all the classical marxian properties.

Theorem of marxian equalization.

1. *For any structure of output, there exists one and only one capitalist reallocation of value.*
2. *If the numeraire is such that the sum of values is equal to the sum of prices for the social net output, then the sum of surplus-values is equal to the sum of profits.*
3. *The rate of profit is a function of the rate of surplus value, of the technical composition of capital in various sectors, and of the allocation of social labor into the sectors, thus of the structure of the output.*

Proof.

1. We are to demonstrate :

$$\forall_y \; \exists (\hat{p}, \gamma) \quad : \quad \begin{cases} \hat{p}.y = v.y & (H_1) \\ \hat{p} = \gamma\,(\hat{p}\,A + w\,\ell) & (H_2) \end{cases}$$

Let us use (H_2) under the form (H_2'). Then $\hat{p}$ is a continuous increasing function of γ, and, thus, so is $\hat{p}.y$ (recall y is a non-negative vector).

When $\gamma = 1$, then : $\hat{p}.y = \frac{1}{1+e}\; v.y < v.y$

When γ tends to the ray of convergence of the series in (H_2'), then $\hat{p}.y$ tends to infinity.

So there is one and only one value for γ, and one and only one covector $\hat{p}$, so that (H_1) holds. That value of γ being more than 1, let it be denoted $\gamma = 1+r$, r being the positive rate of profit (44).

2. Let y be the net output in which the added value is embodied, and Y the corresponding gross output. By definition :

$$\hat{p}.y = v.y \qquad \text{and} \qquad Y - A\,Y = y$$

Then : sum of profits $= \hat{p}.y - w\, \ell.Y$

$$= v.\ (Y - AY) - w\, \ell.Y$$

$$= e\, w\, \ell.Y$$

= sum of surplus value

3. $r = \dfrac{\text{sum of profits}}{\text{engaged capital}} = \dfrac{e\, w\, \ell.Y}{(\hat{p}\, A + w\ell)\, Y} = \dfrac{e}{\dfrac{\hat{p}\, A\, Y}{w\, \ell.Y} + 1}$

Now let us write :

$$\frac{\hat{p}\, A\, Y}{w\, \ell.Y} = \sum_j \frac{\hat{p}\, A_j}{w\, l_j} \cdot \frac{l_j\, Y_j}{\ell.Y}$$

That is the barycentric mean of the organic compositions of capital in various sectors, evaluated in prices of production (45) and weighted by the share of total social labour allocated to sectors. These organic compositions depend only upon the technical composition of the sectors in one hand, and covector $\hat{p}$ on the other hand. But that one depends itself on e, y, and on (A, ℓ). So r depends only on e, y, and (A, ℓ).

Q.E.D.

The last formulation for r is not very suitable, but is the nearest to Marx's formula (the author himself recognized it to be a proxy (46)).

Now let us compute the direct formula, *à la* Duménil-Roy.

Let us start with : $\hat{p}.y = v.y$

Using the (H_2'') formulation and w (1+e) = 1 :

$$w\, \ell\, [\frac{I}{1+r} - A]^{-1}\, y = (1+e)\, w\, v.y$$

hence : $$e = \frac{\ell}{v.y}\, [\frac{I}{1+r} - A]^{-1}\, y - 1$$

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That depends only on the structure of y. So let us denote y^* the vector of same direction, so that : $v.y^* = 1$.

Then : $$e = \ell \left[\frac{I}{1+r} - A\right]^{-1} y^* - v.y^*$$

But : $$\ell = v\ (I-A), \text{ so :}$$

$$e = v\left[(I-A)\left(\frac{I}{1+r} - A\right)^{-1} - I\right] y^*$$

$$= v\left[\left(\frac{r\,I}{1+r} + \frac{I}{1+r} - A\right)\left(\frac{I}{1+r} - A\right)^{-1} - I\right] y^*$$

Finally : $$e = r\ v\ [I - (1+r)\ A]^{-1} y^*$$

That is a continuous increasingfunction of r. So we have a function :

$$r = f_{y^*}(e)$$

This function is formaly the same as in the second part, with y^* in place of d^*. So, in the same way, the corresponding curves are increasing, convex, admit an asymptotic limit $r \to R$, and a finite slope at the origin (47). The enveloppe of the family of curves is composed of arcs of peculiar curves (the ones corresponding to the y^*_j maximizing and minimizing the organic composition at a given r). The reader can report himself to the figure in the Second Part, third paragraph.

III - COMPARISON BETWEEN THE TWO SOLUTIONS.

With the "new" solution to the transformation problem (let us call it henceforth the "solution B") we have obtained all the results that Marx was awaiting from his transformation from value to prices of production. Thus could the results provided by the Morishima-type of solution (let us call it henceforth "solution A") be false ones ? Of course not : they are correct, mathematically speaking, and, economically, they fit with the marxian theory of value and exploitation. And if, when the transformation is performed according to solution B, the workers are enough sheep minded to choose the same consumption bundle d, then the results obtained through solution A are exactly true. Yet they look different from those of solution B !

In fact, there is no contradiction, not even a "dialectical" one. The point is just that neither e nor w have the same meaning nor measure in the two solutions, though they are index numbers of the same reality.

In solution A, w_A is the labour value of the goods purchased by workers :

$$w_A = v.d$$

In solution B, w_B is the share of their value added that they recover and are allowed to spend against commodities according to realloted values :

$$w_B = \frac{1}{1+e_B}$$

There is no reason that $w_A = w_B$, $e_A = e_B$. In the more general case, the embodied value within the goods purchased by wages is not the share of added value reassigned to workers in exchange of their labour power :

$$w_A = v.d \neq w_B = \frac{1}{1+e_B}$$

When the transformation is performed according to solution B, for a given y, does it exist a peculiar structure $\hat{d}^*$ of worker's consumption so that $w_B = v.\hat{d}$? The answer is obvious, through the Second Part of this paper : it is the structure of consumption d^* so that y turns out to be on the integral accumulation model $\hat{y}$ ($\hat{d}^*$). In any other situation, the two solutions are separately valid, with $e_A \neq e_B$. Since the two solutions must deliver the

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same rate of profit r, e_A and e_B are thus connected :

$$e_A = (f^{-1}_{d^*} \circ f_{y^*})\ (e_B)$$

All these results are summed up in the synoptic table.

The solution B being closer both to the intuitions and to the text of Marx, is the solution A, fruit of a long-range amount of serious works, to be surrendered to the museum of curiosities of the history of economic thought ? Not at all, in my opinion. Because it has compeled us to explore carefully (as I tried to do in my Second Part) the economic conceptual context of the transformation : namely, all the problems connected with the "realisability" of the pair (y, d). On the contrary, the "new" solution (in fact, the corrected Marx's one), just because of its simplicity, does not even take into account the necessity for y to be "realized", to be in accordance with some balanced model of accumulation. All that we are winning by expressing directly $r = f_{y^*}(e)$ is compensated by the apparently total undetermination of y^*.

Let us take an example (48). Let w_B be the share of value added recovered by workers. With it, they hasten to purchase the necessary goods and, if possible, some superfluities. But here happens a surprising phenomenon : the production is reoriented (y changes), thus the system of production prices deviates, and the boundary of purchasable bundles of consumption goods changes ! So, for the same rate of exploitation e_B, the same value of labour force w_B, the workers could afford either the necessaries plus some superfluities, or not even the necessaries, depending on the general orientation of production ! That does not fit well with marxist common sense...

In the solution A, we have on the other hand the following statement : "given w_A the value of labour power, the rate of profit varies according to orientation of workers consumption d^*". It is a less shocking statement, in my opinion (49). It is just a transformed and enreached form, within the transformation, of the Volume I theory of "relative surplus value". Marx denotes with that last concept the variation of the rate of surplus value connected with the variation of the value of the bundle d, following the variations in industrial productivity. Now let us assume a change in technology so that the value of d remains the same, but the technical composition

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SYNOPTIC TABLE

Solution A		Solution B
	v = value = embodied labour (theory of value) e = partition of value added (theory of exploitation)	
This partition e_A is defined by embodied labour in workers' consumption bundle d. $w_A = v.d$ defined a priori		This partition e_B is defined a priori, and workers use their share in purchasing commodities at realloted value p $v.d$ defined after the transformation.
	r and p unique when :	
d is given $r = f_{d^*}(e_A)$ r varies according to d^* r does not vary according to the structure of total output		y is given $r = f_{y^*}(e_B)$ r varies according to y^* r does not vary according to the structure of workers' consumption
	Let us choose the *numéraire* so that Σ prices = Σ values	
Σ profits ≠ Σ surplus values (except for $y = \hat{y}(d^*)$) but : Σ values of uses of profit = Σ surplus-values		Σ wages ≠ Σ values of consumption goods (except for $d = \hat{d}(y^*)$) but : Σ profits = Σ surplus values

of industries of consumption goods is modified in a more capitalistic way. There is no relative surplus-value, yet it is intuitive that the general rate of profit must vary (and be very likely lower). That result is made explicit in solution A by the statement "for a fixed value of labour-power the rate of profit depends on the choice of d^*, directed to more or less capitalistic industries" (50).

In post-war capitalism, characterised by a tight connection between labour-to-capital substitution in one hand, and extension of the "consumption society" to the working class on the other hand, all that for reasons of "realisability" of growing productivity (phenomenon that Antonio Gramsci denoted by "fordism"), the structures of y and d are closely connected by complex dynamical processes (51). Here is the reign of the refered dialectics :

$$\begin{array}{ccc} e & \rightarrow & w \\ \uparrow & & \downarrow \\ w & \leftarrow & d \end{array}$$

In such conditions, one must be able to work alternatively with solutions A and B.

But here we enter the domain of capitalism dynamics. Here are set the far more serious problems of the contradictory tendancies of the rate of profit, of overproduction crisis, of inflation... Vital problems which Marx and his main successors have prefered to study in priority, even though overlooking the rather technical problem that P.A. Samuelson may be right, after all, to denote "the so-called transformation problem".

A. LIPIETZ
CEPREMAP
March 1979

FOOTNOTES

1) In 1899 already (!), Antonio Labriola had to reply this kind of attempt (See his "About the crisis of marxism" [9]).

2) See his Marx's economics [12]. I am not at all cancelling the memory of the works of Medio, Meek, Okishio, Seton, Sweezy, Roubine, Winternitz, etc... I have just choosen a well-known reference quite typical of the refered type of solution. For a survey of the problem, from Marx to Von Bortkiewicz, see Dostaler [5], and from von Bortkiewicz up to now, see P.A. Samuelson [16] and Benetti-Cartelier [3].

3) See [16], p.422.

4) I think in particular of Salama [15] and Yaffé [17] as representing the first attitude. For the second one I refer to C. Benetti and J. Cartelier [3].

5) The foundations of this solution were recently laid by G. Duménil [8]. I am very debtfull to conversations with this author, though my arguing is rather different.

6) The following extremely short summary raises many problems which are discussed (refering to Marx's text) in my book [10]. Here it is just a matter of cross-examining the signifiance of the formalism used about the "transformation problem".

7) Vectors y (resp. : linear forms or covectors v) will be denoted by italic letters, or by column-matrix of roman coefficients, on the right side of operations (resp. : by row-matrix on the left side).

8) By "standard productive operation", one understands the normal operation of production according to norms defined by the state of productive forces. It is characterised by a technique, of which are given the quantities of means of production required, and the time needed for its operation. See M. Aglietta [1], A. Lipietz [10].

9) Notice that v and ℓ are consubstantial : abstract labour, not commodities (they are covectors). Once v is computed, one ought to "renormalize" the matrix A by choosing for units of goods the quantities of value one. Whatever may be the unit of value, the coefficients of A are henceforth pure scalars (not quantities of i by unit of j).

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10) I think of C. Benetti, J. Cartelier, J. Fradin. See for instance C. Benetti, "The genesis of the theory of the reproduction - circulation of value", [4], § 24.

11) See Lipietz [10].

12) See Duménil [6].

13) See Marx [11], Volume I, chapter X.

14) Let m be the n-uple m_j , m_j being the quantity of man-power which must be purchased in order to produce one unit of j. Obviously this n-uple is a linear form on vectors y. A 1-covariant 1-contravariant tensor maps it into the covector ℓ.

The difference between ℓ and m is both qualitative (it is not the same linear form) and generally quantitative (it is obvious when m_j is to be measured by "workers-days", and l_j by embodied hours). If we want to explicit the tensor T, it will be denoted in matrix notation $T = \varepsilon\lambda I$ (I being the unity matrix). If ε and λ are not equal in various industries, then we could define ε_i, λ_i, and $T^i_j = \varepsilon_i \lambda_i \delta^i_j$. Note that the same remark could be done about w. If this fact were to be taken into account, e would become a "tensor of exploitation".

This sophistication would be of no interest. Of course, as K. Marx recognizes it quite willingly, there exists differences in the rate of exploitation, and Engels notes that theses differences are likely to be greater than the differences in the rate of profit, for the "equalizing forces are stronger here than there" (see [10], p.268). But :

* First, it is not the point. The crux in the "transformation problem" is that, within the pure theory of value, the rate of profit and the rate of surplus-value could not be both homogenous, when the compositions of capital are different in various sectors. Yet, the "homogenity" of these two rates is implied by the level of abstraction which is ours : all the members of one class are equal, in front of the other class (so the rate of exploitation is homogenous) and in front of the other members of their own class (so the rates of wage and of profit are homogenous).

* Now, if we want a better conceptualisation of concrete reality, we may take into accounts heterogeneities. But why privilege <u>sectorial</u> heterogeneities? In fact, as far as the conditions of exploitation are non-homogenous, the differences lay mainly on sex, race, region, and so on.

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15) The crux of critisisms from C. Benetti and J. Cartelier to the solution of the Morishima-type turns around this idea. But, instead of expliciting the existence of this tensor, these authors just refuse to face the problem of the connection between value and relative prices of commodities. Yet, once things are made clear, one may use the "commandable labour" (here : w m) as an index for "embodied labour" (ℓ). That is what Marx does all along Volume III (see footnote 40).

16) See [11], tome VI, p.191, in French Editions Sociales (provisory quotation).

17) Ibidem, p.196.

18) On the introduction of fixed capital, see [12] and many others... In the same way, we shall assume in this text that the matrix M is indecomposable, and so on.

19) On all that, see Marx [11], Volume III, chapiter IX, in particular p.177 sq. (in french edition).

20) See Nikaïdo [13].

21) Without any assumption about the structure of accumulation, the only constraint is the total realization of output. But capitalists can choose any improductive consumption bundle C, in a n-multiplicity. The inversion of matrix M provides no lowering of the dimension of the multiplicity of corresponding gross output y.

22) See [12] and [16]. We shall refer later to a much more interesting result.

23) See [13].

24) The direction of p being elsewhere fixed, the "normalization" consists in choosing a numéraire so that $\hat{p}.\hat{y} = \nu.\hat{y}$. The divergence from ν to $\hat{p}$ thus depends only on d (for A, ℓ given). On the contrary, the "normalization" of y consists just in selecting the direction $\hat{y}$. Of course, one can completely normalize by choosing also the norm of $\hat{y}$.

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25) Using : $p = (1+r)\,[p\,A + p.d\,\ell]$
and : $v = \ell\,[I-A]^{-1}$

G. Duménil and C. Roy compute :

$$e = r\,v\,[I - (1+r)\,A]^{-1}\,d^*$$

(I shall explicit the computation, for another context, in the third part). d^* being element of the simplex of vertices d_i^* $\left((d_i^*)^j = \delta_i^j\right)$, and, for a given value of r, e being the value of a linear form on d^*, then e is within the interval of extreme values of $f^{-1}_{d_i^*}(r)$. Thus it is sufficient to draw the peculiar curves corresponding to vertices in order to get the enveloppe of all the family of curves (these peculiar curves happen to intersect).

G. Duménil exhibits another more "classical" expresion for f. Let $\bar{y}$ be the vector of activities whose net output is d (namely : $\bar{y} = (I-A)^{-1}\,d$).

Then : $r = e\left[\dfrac{p\,A\,\bar{y}}{p.d\,\ell.\bar{y}} + 1\right]^{-1}$

That is the classical marxian formula $r = \dfrac{e}{q+1}$ (q being the weighted mean of organic compositions), but the organic compositions are valued at prices of production, and the aggregation is applied with the peculiar $\bar{y}$. This non-obvious result allows us to explicit which curves are to compose the enveloppe for a given r.

26) Here are the main lines of J. Roemer's proof. For a total consumption vector $\mathcal{D}$, satisfying the constraint on e, corresponds a covector of prices p according to the Perron-Frobenius Theorem (daily wage is chosen as *numéraire*). For this covector p corresponds a total consumption $\mathcal{D}'$ (by the choice functions). That one can be reduced by similarity to $\mathcal{D}''$ satisfying the constraint on e. The mapping $\mathcal{D} \to \mathcal{D}''$ is continuous on a compact convex. From the Fixed Point Theorem (see [13]), there exists $\mathcal{D}'' = \mathcal{D}$.

27) The interesting point of J. Roemer's formalisation is that the result of class-struggle is summarized by the data e, not d. But, as Reomer himself recognizes :

> *"it may be argued that one should wish the social rate of exploitation to emerge simultaneously with the price-commodity bundle configuration : that is, the "class struggle" over exploitation does not take place in an abstract world,*

but is mediated through the process of exchange as well as at the point of production. This is a reasonable position, and consequently I do not wish to overstate the case for the model presented here as the final solution". [14], p.40.

28) Namely : det $[\lambda I - M] = 0$

29) [16], p.418.

30) [16], p.400

31) The idea that the "natural" or "technical" productiveness of matrix M is at the origin of surplus or profit is an old story, that Marx had already to criticize, against the post-Ricardians such as J.S. Mill.

"*Favourable natural conditions can provide in themselves only the possibility, never the reality of surplus labour, nor, accordingly, the reality of surplus-value and a surplus product. These conditions affect surplus labour only as natural limits, i.e. by determining the point at which labour for others can begin. In proportion as industry advances these natural limits recede. In the midst of our Western European society, where the worker can only purchase the right to work for his own existence by performing surplus labour for others, it is very easy to imagine that it is an inherent quality of human labour to furnish a surplus product. But consider, for example, an inhabitant of the islands of the East Indies, where sago grows wild in the forests. (...) [The Sago is a tree that can be cut off and eaten like bread].*
Suppose now that an East Indian bread-cutter of this kind requires 12 working hours a week for the satisfaction of all his needs. Nature's direct gift to him is plenty of leisure time. Before he can apply this leisure time productively for himself, a whole series of historial circumstances is required ; before he spends it in surplus labour for others, compulsion is necessary. If capitalist production were introduced, the good fellow would perhaps have to work six days a week, in order to appropriate to himself the product of one working day. In that case, the bounty of nature would not explain why he now has to work six days a week, or why he must provide five days of surplus labour. It explains only why his necessary labour-time would be limited to one day a week. But in no case would his surplus product arise from some innate, occult quality of human labour" ([11], Vol. I, chap. XVI, p.650).

32) Recall that ℓ denotes the quantities of labour time necessary in standard productive operations. But these quantities may require ℓ, 2ℓ, 3ℓ days of waged work, according to ε and λ!

33) It is admitted that production lowers by 1 per cent when duration of labour lowers by 2 per cent. On the other hand, at the time of C. Dickens or even J. London, the intensity of labour could suffer out of the bad physical conditions of workers. But, at least in western industrial countries, *that is past time* !

34) P.A. Samuelson recognizes this quite willingly ([15], p.422). Thus it is astonishing that he accepts the solution of F. Seton (which applies on a physical bundle) as a good solution to Marx's transformation problem.

35) That is a decisive remark to understand the inflationist form of present crisis. See [1], [2], [10].

36) The main credit of the discovery of this solution is due to G. Duménil who made clear [7] the two fundamental conditions :
(i) to define w as a share of value added ;
(ii) to apply the transformation onto the net product.

His way of arguing lays on a deep understanding of Marx's approach in the *Capital* (see his book [5]). Yet I take upon myself the exposition in this paragraph, fitting better to my own approach [10], and the necessity and demonstration of the next paragraph.

37) Of course, the least costly technique could depend on e and r - and, worse, a technique could be less costly in value and more costly in prices than another. But here we are not dealing with dynamics.

38) At any stake, the commodity "labour power" could not be identified (as in P.A. Samuelson) with the "output of sector 0", for, in the "firms" of this industry, the labour (the wife's labour) would be unpaid, and the "boss" would sell the output without marking-up the profit. Things would be different if workers would live in capitalist boarding houses, but then the price of labour-power would be $p_0 = (1+r)\ (p.d + p_0\ l_0)$, l_0 denoting the labour of the staff of the boarding house.

39) The "choice functions" are surely not, as in Roemer, of the neo-classical type. They are rather of the lexicographical type : first one chooses the necessaries, then more and more superfluities (the ordering of necessaries and superfluities being socially defined).

40) The "commended" or "commendable" labour is a bastard concept of Classical Economics (A. Smith, D. Ricardo), which K. Marx uses again explicitly in Theories of surplus-value and implicitly in Volume III of *Capital*. It denotes the value or the price of purchased manpower, when used as index of delivered labour. In other words, it is variable capital V as for as it may be used as index number for the quantity of value added V+PL (w and T being implied). For instance, when Marx denotes C/V the composition of capital in Volume III, he outlines that here V is used as index for embodied labour. But many authors, who did not notice that, are puzzled : they cannot understand how the increasing of C/V could imply the fall of the rate of profit. Yet, once Marx's words are understood (namely, that the "growth of organic composition" is in fact the growth of $\frac{C}{V+PL}$), it becomes obvious that the rate of profit tends towards 0 uniformly (that is : whatever could be the evolution of e = PL/V) with the growth of organic composition of capital. (On this little trap, see [10]).

41) The quantity of labour directly embodied in j is l_j ; the quantity of direct labour embodied in the means of production of j is ℓA_j ; in the means of production of these means of production : $\ell [A^2]_j$ and so on... T being the unity tensor, one gets the "commended labour" in multiplying by w.

42) The series in H_2' converge towards the inverse matrix when $\gamma < 1+R$, $\frac{1}{1+R}$ being the root of Frobenius of matrix A. That condition holds, as it will turn out, when workers have a non-zero consumption, matrix A being itself productive (see [13]).

43) This infinite regressive splitting of the "constant capital" value between past "variable capital" and past surplus value (or : of the price of the means of production, between wages and profit) has already been performed by Adam Smith. We must assume a fictive genealogy, with constant norms of production in the past. Since we are splitting the value of the *net* product, the sums of "variable capital" and "surplus-value" resulting of the splitting are equivalent to the respective shares of workers and capitalists within the value added in the period.

44) So $r > 0$ iff $w < 1$ or $e > 0$. It is the result of the "fundamental marxian theorem", in the new context.

45) Of course, these "technical" coefficients a_j^i and l_j are the expression of social relations : division of labour, taylorism, fordism, and so on. By "technical", I only mean that the coefficients are given "before" the conditions of extortion of surplus value $(w, \lambda, \varepsilon)$, and in particular are independent of d. So I intend to outline the difference with the Morishima-type of solution.

46) In Marx's proxy, where $r = \frac{\Sigma\, PL_i}{\Sigma(C_i+V_i)}$, the organic compositions are estimated in value. *That is the only consequence of Marx's proxy.*

47) By expanding in series the computed expression of $e = f_{y^*}^{-1}(r)$, it becomes obvious that : $\forall\, r$, $\frac{d\, e}{d\, r} > 0$, $\frac{d^2\, e}{d\, r^2} > 0$, and $r \to 0 \Rightarrow \frac{e}{r} \to \nu\, (I-A)^{-1}\, y^*$.

48) Here is another one, that I shall not develop, because it implies the matrix M to be decomposable, and this would make the paper still longer. It is well known by practicians of solution A that, if there exists non-fundamental sectors, the rate of profit is determined by the sub-section of economy of fundamental goods. That result looks shocking for many marxists : the general rate of profit would not be influenced by the surplus-value produced in the "section III" (the section of non-fundamental industries). The intuition underlying to their criticism is that, assuming a weak composition of capital in section III, it would be sufficient to increase the share of social labour alloted to this section in order to increase the general rate of profit. That intuition holds on the apparently total undetermination of vector y in solution B. But that is a misleading appearence. The output of non-fundamendal sectors is to be realized, and, except for the auto-consumption by capitalists of this section, it is purchased by profits from the other sections. Thus, the non-fundamental component y_{NF} of y is a function of the components $\hat{y}_F$ and $\tilde{y}_F$ (defined in Part Two). When taking these constraints into account, one rediscovers the results of solution A. Yet that solution directly exhibits the result : "once given the workers consumption bundle d, for any *realized* output (that is : produced *and* purchased), the rate of profit is constant". And it is a pretty result...

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49) Quite a subjective opinion, of course. For G. Duménil, it was such an unacceptable result that it led him to discard the solution A and approach the solution B.

50) Here I seem mixing dynamics and comparative statics. But it can be assumed that there are two substitutable consumption goods of same value but with quite different technical coefficients. The "choice" from one to the other good by the working class will have the same effect on the rate of profit as a change in technology.

51) These processes are the main characters of what I call "the monopolistic regulation of intensive accumulation" [10].

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