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TAXING PRICE MAKERS

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INTRODUCTION.

It has been early recognized that the existence in the production sector of distortions between marginal rates of transformation and relative prices might invalidate the traditional conclusions of welfare economics (see Lipsey and Lancaster (1958) for a defense of this argument). Actually, part of the second best literature is aimed at elucidating the normative implications of the presence in the production sector of distortions which originate in policy constraints (Boiteux (1956)), in fixed taxes (Green (1962), Sandmo and Drèze (1971)) or in some degree of monopoly power (Bergson (1972)).

In all of these studies, producers respond to prices only, and their behavior can be interpreted as a "deviant" or non competitive behavior with respect to some price vector and formalized through a correspondence which associates to this vector a set of production plans (as in Mc Fadden (1969) or Guesnerie (1975)). Clearly, this formulation cannot apply to price makers, in particular to genuine monopolists whose supplies depend in general on other variables in addition to prices.

Even though distortions created by monopolies were one of the initial motivations of this literature, the study of price makers in a second best framework remains rather poor. The conventional wisdom is that in an economy with price makers it is possible to restore first best Pareto efficiency with appropriate taxation schemes. This "tradition" seems to rest on partial equilibrium formulations of the problem or on general equilibrium models (Bohm (1971)) which do not recognize the full complexity of the problem.

Our work is an attempt at a comprehensive treatment in a general equilibrium framework of the question of decentralization of Pareto optima in economies with price makers where linear taxes are available. The strategic behavior of price makers is formalized through a reaction correspondence which in the case of the monopolist has an easy interpretation when he is assumed to know the true demand function. New results stem both from the general equilibrium approach and from the explicit recognition of a fundamental non convexity.

In section I, the essential discontinuity of the monopolist's reaction function is explained. Then, our basic model and assumptions are discussed. Adopting successively the second best theory point of view and the decentralization point view, sections II and III develop our theory of optimal taxation of a price maker when, in addition to linear taxes, lump sum transfers are available policy tools. We first show that, in a regular case, a linear excise tax on the price maker (which turns out to be a subsidy in the case of a normal good) enables the Government to restore first best Pareto efficiency. However the non regular case is shown to be non pathological. Indeed an example of a simple economy where a first best Pareto optimum can never be attained is given. The non regular case presents a noteworthy property : the corresponding second best situation is generally highly unstable ; a small variation of the environment or a small error in the choice of policy tools induce the price maker to choose a production level which is far from the optimal one. In some sense the optimizing behavior of the Government tends to lead the system, which would have been otherwise stable, to unstable situations("catastrophic" points). This latter feature is closely related to the fact that the maximization program of the agent that the Government attempts to control is not concave. An analysis similar to this one would be relevant in any control problem involving an agent which has a reaction function discontinuous with respect to his environment (this point has been stressed independently by Mirrlees in Mirrlees (1977)). Section IV extends the analysis to the case where lump sum transfers are not available. Then, distributional and allocational goals become conflicting. The difficulties created by these types of question are explored under several distributional assumptions, the Diamond-Mirrlees case where profits are 100 % taxed and cases where profits are only partially taxed. Then, even in the regular case, first best Pareto optimality is not in general achievable. The characteristics of the non regular case are extended to this situation. Finally some comments on the informational assumptions underlying our work are offered.

I - PRELIMINARIES.

I.1 - A MAIN DIFFICULTY : DISCONTINUITIES IN THE MONOPOLIST'S REACTION FUNCTION.

The non concavity of the profit function of a monopolist is now a well known phenomenon. Such a non concavity induces discontinuities in the monopolist's response to changes in the price system and in the parameters of the economy. These discontinuities are the main obstacle to the integration of monopoly theory in a general equilibrium setting of the Arrow-Debreu type ; the need for ad hoc assumptions to obtain convex valued supply correspondences has been the major teaching of repeated attempts by Arrow (1971), Arrow and Hahn (1971), Fitzroy (1974), Gabszewicz and Vial (1972), Negishi (1961), Laffont and Laroque (1976), Marschak and Selten (1974) to prove the existence of a monopolistic general equilibrium. Recently, Roberts and Sonnenschein (1977) have told more explicitly the same story by proving the possibility of the construction of an economy without any monopolistic equilibrium.

Beyond the existence question, this discontinuity in the monopolist's reaction function has also a serious influence on the normative analysis presented here, by requiring special attention to those critical points where marginal changes create non marginal consequences. So, as a preliminary necessary work, we focus our attention on this problem in the remainder of this subsection.

Consider an economy with L + 1 commodities. Good 0 is supplied by a monopolist. Goods 1,...,L are supplied by competitive firms, and competitive consumers demand all of the goods. Let R_i , $i = 1,\ldots,I$, be the income of consumer i and let $R = (R_1,\ldots,R_I)$. The consumer price vector is denoted $\Pi = (\pi_0,\pi) \in \mathbb{R}_+ \times \mathbb{R}_+^L$ and the producer price vector is denoted $P = (p_0,p) \in \mathbb{R}_+ \times \mathbb{R}_+^L$. Let $T = \Pi - P = (t_0,t) \in \mathbb{R} \times \mathbb{R}^L$ the vector of taxes. Finally let $\xi_0(\Pi,R)$ be the continuous demand function for good 0 and let $C(q,p)$ be the continuous cost function of the monopolist firm.

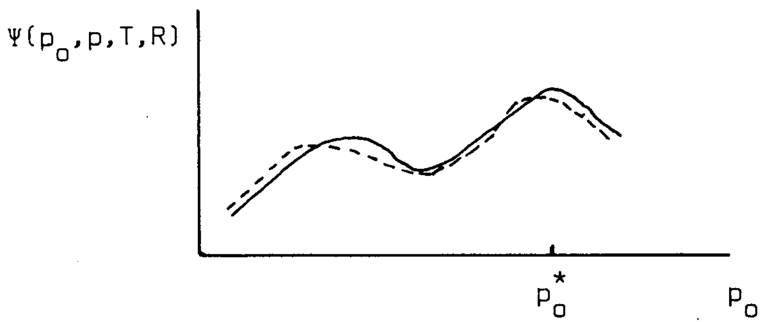
The profit function of the monopolist is therefore :

$$\Psi(p_o,p,T,R) = p_o\xi_o(P+T,R) - C(\xi_o(P+T,R),p)$$

$$= p_o\xi_o(p_o+t_o,p+t,R) - C(\xi_o(p_o+t_o,p+t,R),p)$$

The function $\Psi(.,p,T,R)$ is in general non concave and we have essentially two cases. (see figure 1)

<u>Case 1</u> : (unique maximizing price)



<u>FIGURE 1a</u>.

In figure 1a, p_o^* is the unique maximizing price of the monopolist. Moreover, as a function of the environment (p,T,R), p_o^* is a continuous function, since a marginal change of these variables, leading to the dotted curve, induces only a marginal change of the monopolist's optimal price choice.

Case 2 : (multiple maximizing prices)

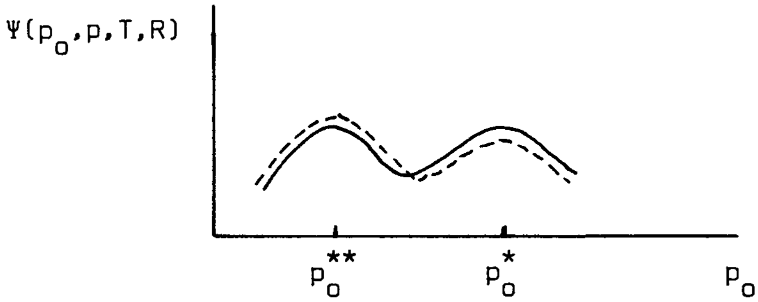


FIGURE 1b.

In figure 1b, two maximizing prices exist for the monopolist. Suppose p_o^* is chosen. Then, a marginal change of (p,T,R) (leading to the dotted curve) creates a large discontinuity in the behavior of the monopolist for whom a value close to p_o^{**} becomes optimal.

Generally speaking, the maximization of his profit function by the monopolist determines a price correspondence $\mathcal{P}_o$, which associates to every environment (p,T,R) a set of optimal prices. Assuming that demand functions are single valued, let Q_o be the induced quantity correspondence. These correspondences are multivalued and non convex valued at critical points corresponding to case 1b, and they are single valued at regular points corresponding to case 1a.

Note that the monopoly choice problem can be approached with the quantity instead of the price as a decision variable : If $\pi_o(q,\pi,R)$ is the inverse demand function for the monopolized commodity, the profit function of the monopolist is :

$$\Phi(q,p,T,R) \quad = \quad [\pi_o(q,\pi,R) - t_o]\, q \quad - C(q,p)$$

$\Phi(.)$ is non concave in q when $\Psi(.)$ is non concave in p.

Having made explicit the discontinuity of the monopolist's reaction function, we consider in the next sections the more general case of a price maker agent, defined by his price correspondence $\mathcal{P}_0(p,T,R)$.

This formulation may be used in the description of a wide variety of oligopolistic competitions. In particular in the case of a Cournot oligopoly $\mathcal{P}_0(p,T,R)$ is the set of agreed prices obtained as Nash equilibria of the associated game.

I.2 - GENERAL FEATURES OF THE MODEL.

We consider an economy with the following physical characteristics. There are L + 1 commodities indexed by $\ell = 0,\dots,L$. Commodity 0 is a final commodity. Consumers are indexed by $i = 1,\dots,I$. Production sets Y_j , $j = 0,\dots,J$, are associated with private producers. Y_g denotes the production set of the public sector. The general assumptions on preferences and technology are the following.

Assumption 1 :

Consumer i has a monotone strictly concave utility function $u_i(.)$ defined on a convex consumption set X_i , $i = 1,\dots,I$.

Assumption 2 :

The production set Y_j of producer j , $j = 0,\dots,J$ is convex. In addition the production set Y_g of the public sector defined by $f(y_g) \geq 0$ is convex.

In addition commodity 0 is produced only by firm 0.

The set of feasible states of this economy given the technological constraints, which we call the set of *physical feasible states,* is defined as follows :

$(x_i)(y_j)(y_g)$ is a physical feasible state iff :

(1.1) $\quad x_i \in X_i \quad , \quad i = 1,\dots,I \ , \ y_j \in Y_j \quad , \quad j = 0,\dots,J \quad , \quad y_g \in Y_g$

(1.2) $\quad \sum_{i=1}^{I} x_i \leq \sum_{j=0}^{J} y_j + y_g$

In the whole paper we suppose that :

. Firm 0 is a price maker on the market of commodity zero and has a competitive cost minimizing behavior on the other markets. All the other agents have a competitive behavior on all markets.

. Consumers and producers are faced with two different price systems, denoted as in the preceding section $\Pi = (\pi_0,\pi)$, $P = (p_0,p)$. ($T = (t_0,t)$ and $R = (R_1,\dots,R_I)$ denote taxes and incomes).

Assumptions 3 and 4 are regularity assumptions concerning respectively demand and supply functions and the cost function of the price maker. Assumptions 5 and 6 are local regularity assumptions.

Assumption 3 :

The utility functions $u_i(.)$, the indirect utility functions $V_i(\Pi,R_i)$ and the demand functions $\xi_{i\ell}(\Pi,R_i)$ of agent i , $i = 1,\dots,I$, the supply functions $\eta_{j\ell}(p)$ of the competitive producers $j = 1,\dots,J$, the production function $f(.)$ of the public sector, are continuously differentiable in the interior of their domains.

$$\text{Let} \qquad \xi_\ell(\Pi,R) = \sum_{i=1}^{I} \xi_{i\ell}(\Pi,R_i) \qquad , \quad \ell = 1,\dots,L$$

$$\eta_\ell(p) = \sum_{j=1}^{J} \eta_{j\ell}(p) \qquad , \quad \ell = 1,\dots,L$$

Assumption 4 :

Let q be the production level of commodity 0 ; the cost function of the price maker $C(q,p)$ and his net supply on markets 1 to L, $x_\ell(p,q)$ are continuously differentiable in the interior of their domains.

Assumption 5 :

The (L,L) matrix A defined by

$$a_{\ell k} = \left(\frac{\partial \eta_\ell}{\partial p_k} - \frac{\partial x_\ell}{\partial p_k}\right) \qquad \begin{array}{l} \ell = 1,\dots,L \ , \\ k = 1,\dots,L \end{array}$$

is of rank $L - 1$.

Assumption 6 :

The $(L + 1 \ , L + 1)$ matrix S defined by

$$s_{\ell k} = \sum_{i \in I} \left(\frac{\partial \xi_{i\ell}}{\partial t_k} + \xi_{ik} \frac{\partial \xi_{i\ell}}{\partial R_i}\right)$$

which is the matrix of aggregate compensated price effects is of rank L.

Given the institutional constraints we have just mentioned a *feasible state of the model considered in this paper* consists of consumption plans x_i , $i = 1,\dots,I$, production plans $y_o = (q,x_o)$, y_j , $j = 1,\dots,J$, y_g , associated with (Π,P,R) such that :

(1.3) $\quad \eta_\ell(p) + y_{g\ell} - \xi_\ell(\Pi,R) - x_{o\ell} \geq 0$

(1.4) $\quad x_{o\ell} = x_\ell(p,q)$

(1.5) $\quad q - \xi_o(p_o + t_o,\pi,R) = 0$

(1.6) $\quad p_o \in \mathcal{P}_o(p,T,R)$

(1.7) $\quad f(y_g) \geq 0$

./.

Eventually, in the models we will consider, additional constraints on T , y_g , R will be considered corresponding to different assumptions on the set of policy tools which are available to the Government.

- If there is no other significant constraint than (1.3) to (1.7), the Government can disconnect freely production and consumption prices, decide upon the production plans of the public sector and control the income of each individual through a program of lump sum transfers. This will be the model considered in sections II and III

- In section IV we will assume a more restricted set of policy tools. Lump sum transfers will be supposed impossible in section IV.1 , i.e $R_i = 0$, $i = 1,\ldots,I$. In subsections IV.2 and IV.3 , non labor income will arise from the distribution of price maker's profit.The assumption of unrestricted commodity taxation will be relaxed in subsection IV.3 .

II - TAXING PRICE MAKERS WHEN LUMP SUM TRANSFERS ARE AVAILABLE : THE SECOND BEST THEORY POINT OF VIEW.

II.1 - THE MAXIMIZATION PROGRAM.

We assume that the Government can realize any distribution of income $(R_1,\dots,R_I)$ through a program of lump sum transfers. This restrictive assumption has been made implicitly in the whole literature on optimal taxation with monopolies. A priori it gives to the Government powerful policy tools for solving distributional problems ; in particular the question raised by the pure profit of the monopolist. The Government may also completely disconnect production and consumption prices with linear commodity taxes and control production in the public sector.

Let us first consider a single price maker. Given a social welfare function defined by the weights $\alpha_i > 0$, $i = 1,\dots,I$, the Government's maximization problem can then be written(1) :

$$\text{(I)} \qquad \underset{(p,T,R)}{\text{Max}} \sum_{i\in I} \alpha_i V_i(\Pi,R_i)$$

s.t.

$$(2.1) \qquad \eta_\ell(p) + y_{g\ell} - \xi_\ell(\Pi,R) - x_\ell(p,q) \geq 0 , \quad \ell = 1,\dots,L$$

$$(2.2) \qquad q - \xi_o(p_o + t_o,\pi,R) = 0$$

$$(2.3) \qquad p_o \in \mathcal{P}_o(p,T,R)$$

$$(2.4) \qquad f(y_g) \geq 0$$

$$(2.5) \qquad p \geq 0 \qquad\qquad \pi \geq 0$$

./.

If the producer of good 0 was behaving competitively we know that the Government could then be able to achieve a first best optimum. With production prices it could control private production locally. With consumer prices and lump sum transfers it could control production locally. In the set of "physical" feasible allocations the government could then ensure a displacement in any direction. Any solution of program (I) would then have to be a local optimum of the social welfare function in the set of feasible allocations. Moreover, in the convex economy we defined, such a local optimum would also be a global optimum, i.e, a first best optimum.

The introduction of a price maker defined by the correspondence $\mathcal{P}_0(.)$ raises the question : under which conditions is a solution of programm (I) still a first best optimum ? Consider such a solution. First in the special case of a monopolist, we show geometrically and hopefully intuitively that if his reaction function is differentiable with respect to the instruments in a neighbourhood of the solution, it is possible to control locally the production vector of the monopolist in addition ot the behavior of any other agent. Consequently as above, we are at a first best optimum. However, because of a fundamental non concavity in the price maker's profit function there are discontinuities in his responses to changes in the parameters. Therefore, it may happen that at the solution of program (I) the reaction function of the monopolist be not differentiable. Then, such a solution is not in general a first best optimum.

II.2 - LOCAL CONTROLLABILITY WHEN THE PRICE MAKER IS A MONOPOLIST : A "GEOMETRIC" INSIGHT.

We consider in this section a simplified version of the economy presented above with one consumer (u,X) , and one public firm (private firms are excluded). The price taker is a monopolist with the profit function given in section 1

$$\Phi(q,p,T,R) = [\pi_0(q,\pi,R) - t_0]\, q - C(q,p).$$

The vector $e = (p,T,R)$ defines the environment of the monopolist. We will see that the Government, using the available policy tools, is able to have local control of the economy on the "physical" feasible set around initial situations where the production of the monopolist is regular.

Let us first define precisely what is a regular production plan of the monopolist. Let $B^d\ Y_o$ be the boundary of the monopolist's production set. The set Λ_o of attainable regular production plans of the monopolist is defined as follows :

$$y_o = (q_o,x_o) \in \Lambda_o \iff \exists \ e = (p,T,R) \text{ such that :}$$

- $Q_o(.)$ is single valued and continuous at (e).
- $q_o = Q_o(p,T,R)$, $x_o = x(p,q_o)$
- the profit function $\phi(.,e')$ is differentiable for any e' in neighbourhood of e.

Λ_o is the set of production plans which are optimal for the monopolist for some state of the environment and in which small perturbations of the environment induce smooth reactions of the monopolist.

Consider then a state $(\overline{y}_o,\overline{y}_g,\overline{x})$ of the simplified economy such that :

$$\overline{x} = \xi(\overline{\Pi},\overline{R}) \quad , \quad \overline{y}_o = (\overline{q}_o = Q_o(\overline{p},\overline{T},\overline{R}),\overline{x}_o = x(\overline{p},\overline{q}_o)),\overline{y}_o \in \Lambda_o \quad ,$$
$$\overline{x} = \overline{y}_o + \overline{y}_g \quad .$$

$\overline{y}_o$, $\overline{y}_g$, $\overline{x}$ is a feasible state of the model associated with equations (2.1) to (2.5) ; it has the additional property that $\overline{y}_o \in \Lambda_o$.

The following lemma states that in this case, the Government, using adequately the policy tools, can induce the agents to choose any 3-uple $y_o \in Y_o$, $y_g \in Y_g$, $x \in X$ close to $\overline{y}_o$, $\overline{y}_g$, $\overline{x}$ *and* meeting the constraints $x = y_o + y_g$. In other words, all agents can be induced to change their demands and supplies in any direction compatible with technological and scarcity constraints.

Lemma 1.

Let $\overline{y}_o = (\overline{q}_o, \overline{x}_o)$, $\overline{y}_o \in \Lambda_o$ associated with $\overline{p}, \overline{T}, \overline{R}$; let $\overline{x} = \xi(\overline{\Pi}, \overline{R})$, $\overline{y}_g \in Y_g$ such that $\overline{x} = \overline{y}_g + \overline{y}_o$. Under A1 to A4 for any $\{y_o = (q_o, x_o), y_g, x\} \in \{V(\overline{y}_o) \cap B^d Y_o\} \times \{V(\overline{y}_g) \cap Y_g\} \times \{V(\overline{x}) \cap X\}$ such that $x \leq y_o + y_g$ (where $V(.)$ are appropriated neighbourhoods of $(.)$) then there exist p, T, Π, R close to $\overline{p}, \overline{T}, \overline{\Pi}, \overline{R}$ such that $x = \xi(\Pi, R)$, $q_o = Q_o(p, T, R)$, $x_o = x_o(p, q_o)$.

PROOF.

It is clear that there exist Π, R close to $\overline{\Pi}, \overline{R}$ such that $x = \xi(\Pi, R)$. Since $y_o \in V(\overline{y}_o) \cap B^d Y_o$, there exists p close to $\overline{p}$ such that $x_o = x(p, q_o)$.

Furthermore

$$t_o = \frac{\partial \pi_o}{\partial q}(q_o, \pi, R)\, q_o + \pi_o(q_o, \pi, R) - \frac{\partial C}{\partial q}(q_o, p)$$

where the right hand side defines for π, R, p close enough to $\overline{\pi}, \overline{R}, \overline{p}$ the optimal tax t_o, close to $\overline{t}_o$ since the right hand side of the above expression is continuous in all the variables, which induces the monopolist to produce q_o.

Q.E.D.

Let us now consider $\zeta^* = (x^*, y_o^*, y_g^*)$ an optimum of our one consumer economy with monopoly, associated with e^* and such that $x^* = \xi(\Pi^*, R^*)$, $y_o^* \in \Lambda_o$.

PROPOSITION :

Let $\zeta^* = (y_o^*, y_g^*, x^*)$ be an optimum of the one consumer economy defined by equations (2.1) to (2.5) and associated with a regular y_o^*. Then, under A1 to A4, on the one hand $X(x^*) = \{x \in X,\ x \succ x^*\}$ and $Y_o + y_g^*$, and on the other hand $Y_g + y_o^*$ and $X(x^*)$ are locally separated.

PROOF.

Let us suppose for example that $Y_g + y_o^*$ and $X(x^*)$ are not locally separated. Then, there exist $x' \in X(x^*)$ such that $x' = y_o^* + y_g'$ for some y_g' close to y_g^*. But the preceding lemma asserts that y_o^*, y_g', x' is a feasible state of our model contradicting the optimality of ζ^*. A similar reasoning applies for $Y_o + y_g^*$ and $X(x^*)$.

Q.E.D.

It follows that, if Y_g and Y_o are convex, we have in addition that :

$$\pi^* \cdot y_g^* \geq \pi^* \cdot y_g \quad \forall\, y_g \in Y_g \ , \ \Pi^* \cdot y_o^* \geq \Pi^* \cdot y_o \quad \forall\, y_o \in Y_o$$

and that ζ^* is a first best optimum.

These propositions are straightforward consequences of lemma 1. A state where the monopolist's production, public production and private consumption can be locally controlled in the sense of lemma 1, can be improved upon in the sense of some social welfare function as soon as it is not a local maximum of the first best problem, (i.e. as soon as it is not a first best optimum because we are in a convex problem). Hence second best optima occuring at regular production plans are first best optima.

However, lemma 1 does not hold when $y_o \notin \Lambda_o$. In this case the Government can only modify the monopolist's production plans in some directions depending on the associated changes in consumption and incomes. Hence the second best allocation chere $y_o^* \notin \Lambda_o$ will not be (generally) first best.

We will see in the next section, that this analysis is general. Two different characterization theorems are obtained according to whether the production plan of the price taker is regular or not. In the first case, first best Pareto optimality is restored. In the second case, optimality conditions are more complicated and shadow prices to be given to public firms are no longer consumption prices.

II.3 - THE REGULAR CASE : GENERAL RESULTS.

We generalize in this subsection the above results to any type of price maker. We observe that a characterization of the maxima of program (I) can be obtained through standard techniques only when at the optimum the correspondence $\mathcal{P}_0$ is single valued and differentiable. Indeed we have :

THEOREM 1.[2]

Let (p_0^*, p^*, T^*, R^*) be an optimal solution of program (I) with $(p_0^*, p^*) \gg 0$. Under A1 - A4, if in a neighbourhood of (p^*, T^*, R^*) , the correspondence $\mathcal{P}_0$ reduces to a single valued and continuously differentiable function such that $\frac{\partial \mathcal{P}_0}{\partial t_0}(p^*, T^*, R^*) \neq -1$, if A5, A6 are satisfied at the optimal solution, then the corresponding state is first best Pareto optimal. Moreover T^* can be normalized so that only commodity 0 is taxed.

PROOF.

The Lagrangien $\mathcal{L}$ of programm (I) is :

$$\mathcal{L} = \sum_{i \in I} \alpha_i V_i(P+T, R_i) + \sum_{\ell=1}^{L} \rho_\ell[\eta_\ell(p) + y_{g\ell} - \xi_\ell(p_0+t_0, p+t, R) - x_\ell(p,q)]$$

$$+ \rho_0[-\xi_0(p_0+t_0, p+t, R) + q] + \lambda[p_0 - \tilde{\mathcal{P}}_0(p,T,R)] + \mu f(y_g) + \nu p + \gamma\pi$$

where $\tilde{\mathcal{P}}_0(p,T,R)$ is a continuously differentiable function which coincides with $\mathcal{P}_0$ in a neighbourhood of (p^*, T^*, R^*).

Monotonicity of preferences and availability of lump sum transfers implies $f(y_g^*) = 0$, $\mu^* > 0$, $\Pi^* \gg 0$. Hence, as $p^* \gg 0$, the Kuhn and Tucker conditions reduce to

$$(2.6) \qquad \sum_{i \in I} \alpha_i \frac{\partial V_i}{\partial p_o} - \sum_{\ell=0}^{L} \rho_\ell^* \frac{\partial \xi_\ell}{\partial p_o} + \lambda^* = 0$$

$$(2.7) \qquad \sum_{i \in I} \alpha_i \frac{\partial V_i}{\partial t_o} - \sum_{\ell=0}^{L} \rho_\ell^* \frac{\partial \xi_\ell}{\partial t_o} - \lambda^* \frac{\partial \tilde{\mathcal{P}}_o}{\partial t_o} = 0$$

$$(2.8) \qquad - \sum_{\ell=1}^{L} \rho_\ell^* \frac{\partial x_\ell}{\partial q} + \rho_o^* = 0$$

$$(2.9) \qquad \sum_{i \in I} \alpha_i \frac{\partial V_i}{\partial p_k} - \sum_{\ell=0}^{L} \rho_\ell^* \frac{\partial \xi_\ell}{\partial p_k} - \sum_{\ell=1}^{L} \rho_\ell^* \left(\frac{\partial x_\ell}{\partial p_k} - \frac{\partial \eta_\ell}{\partial p_k}\right) - \lambda^* \frac{\partial \tilde{\mathcal{P}}_o}{\partial p_k} = 0 \qquad k = 1,\ldots,L$$

$$(2.10) \qquad \sum_{i \in I} \alpha_i \frac{\partial V_i}{\partial t_k} - \sum_{\ell=0}^{L} \rho_\ell^* \frac{\partial \xi_\ell}{\partial t_k} - \lambda^* \frac{\partial \tilde{\mathcal{P}}_o}{\partial t_k} \qquad k = 1,\ldots,L$$

$$(2.11) \qquad \rho_\ell^* + \mu^* \frac{\partial f}{\partial y_{g\ell}} = 0 \qquad \ell = 1,\ldots,L$$

$$(2.12) \qquad \alpha_i \frac{\partial V_i}{\partial R_i} - \sum_{\ell=0}^{L} \rho_\ell^* \frac{\partial \xi_{i\ell}}{\partial R_i} - \lambda^* \frac{\partial \tilde{\mathcal{P}}_o}{\partial R_i} = 0 \qquad i \in I$$

Observing that $\frac{\partial V_i}{\partial p_o} = \frac{\partial V_i}{\partial t_o}$, $\frac{\partial \xi_\ell}{\partial p_o} = \frac{\partial \xi_\ell}{\partial t_o}$ and subtracting (2.7) from (2.6) we obtain

$$(2.13) \qquad \lambda^* \left[1 + \frac{\partial \tilde{\mathcal{P}}_o}{\partial t_o}\right] = 0$$

Therefore, if $\frac{\partial \tilde{\mathcal{P}}_o}{\partial t_o} \neq -1$, we get

$$(2.14) \qquad \lambda^* = 0$$

Observing that $\frac{\partial V_i}{\partial p_k} = \frac{\partial V_i}{\partial t_k}$ and $\frac{\partial \xi_\ell}{\partial p_k} = \frac{\partial \xi_\ell}{\partial t_k}$, we subtract (2.10) from (2.9) and using (2.14) we have :

$$(2.15) \qquad \sum_{\ell=1}^{L} \rho_\ell^* \left(\frac{\partial n_\ell}{\partial p_k} - \frac{\partial x_\ell}{\partial p_k} \right) = 0 \qquad k = 1,\dots,L$$

From the homogeneity of degree zero of $n_\ell(.)$ and $x_\ell(.)$, $\ell = 1,\dots,L$ and the symmetry of the Jacobians of n_ℓ and x_ℓ we have

$$\sum_{\ell=1}^{L} p_\ell^* \frac{\partial n_\ell}{\partial p}(p^*) = 0$$

$$\sum_{\ell=1}^{L} p_\ell^* \frac{\partial x_\ell}{\partial p}(p^*) = 0$$

which implies with assumption A5 :

$$(2.16) \qquad \rho_\ell^* = k\, p_\ell^* \qquad \ell = 1,\dots,L$$

Now, multiplying equation (2.12) by ξ_{ik}^* , $i \in I$ and adding all these equations to the k^{th} equation of (2.10) we obtain :

$$(2.17) \qquad \sum_{i\in I} \alpha_i \left(\frac{\partial V_i}{\partial t_k} + \xi_{ik}^* \frac{\partial V_i}{\partial R_i} \right) - \sum_{\ell=0}^{L} \rho_\ell^* \sum_{i\in I} \left(\frac{\partial \xi_{i\ell}}{\partial t_k} + \xi_{ik}^* \frac{\partial \xi_{i\ell}}{R_i} \right) = 0$$

Since, from Roy's identity

$\frac{\partial V_i}{\partial t_k} = - \xi_{ik}^* \frac{\partial V_i}{\partial R_i}$, (2.17) reduces to

$$(2.18) \qquad \sum_{\ell=0}^{L} \rho_\ell^* \sum_{i\in I} \left(\frac{\partial \xi_{i\ell}}{\partial t_k} + \xi_{ik}^* \frac{\partial \xi_{i\ell}}{\partial R_i} \right) = 0$$

But, since $\frac{\partial \xi_{i\ell}}{\partial t_k} + \xi^*_{ik} \frac{\partial \xi_{i\ell}}{\partial R_i}$ is by the Slustsky equations the derivative of the compensated demand function evaluated at Π^*, we have

$$(2.19) \qquad \sum_{\ell=0}^{L} \pi^*_\ell \sum_{i\in I} \left(\frac{\partial \xi_{i\ell}}{\partial t_k} + \xi^*_{ik} \frac{\partial \xi_{i\ell}}{\partial R_i} \right) = 0$$

then, from A6,

$$(2.20) \qquad k' \pi^*_\ell = \rho^*_\ell \qquad \ell = 0,1,\dots,L$$

which, with (2.16) implies :

$$(2.21) \qquad \pi^*_\ell = \frac{k}{k'} \quad p^*_\ell \qquad \ell = 1,\dots,L$$

From (2.16), (2.20), (2.8) we have :

$$(2.22) \qquad \frac{k'}{k} \quad \pi^*_o = \sum_{\ell=1}^{L} p^*_\ell \frac{\partial x_\ell}{\partial q} = \frac{\partial C}{\partial q}(q^*,p^*)$$

where $C(q,p)$ is the cost function of the price maker firm.

Therefore, the production plan of the price maker would be competitive with respect to the price system $(\frac{k'}{k} \pi^*_o, p^*_1, \dots, p^*_L)$.

Also (2.11) implies with A2 that the shadow prices of the controlled firm are p^*_ℓ , $\ell = 1,\dots,L$, and that $p^* y^*_g \geq p^* y_g$ for any y_g such that $f(y_g) \geq 0$.

It follows that the production sector behaves as if it were competitive with respect to the price vector

$$\tilde{p}^* = (\frac{k'}{k} \pi^*_o, \dots, p^*_L)$$

Similarly, consumers face the price system $(\pi_o^*, \frac{k}{k'} p_1^*, \ldots, \frac{k}{k'} p_L^*)$ which is proportional to $\tilde{P}^*$.

Hence, the economic state $[(\xi_i(\Pi^*, R_i^*)), \eta(p^*), (y_g^*), q^*, (x_\ell^*(p^*, q^*))]$ is an equilibrium and therefore (with A1) a Pareto optimum (cf Debreu (1959)).

Since there is one degree of freedom in choosing the norms of the production price vector and consumption price vector, we can select k , k' such that :

$$\pi_\ell^* = p_\ell^* , \qquad \ell = 1, \ldots, L$$

The only taxed commodity is then commodity 0 and from (2.22)

$$(2.23) \qquad t_o^* = \frac{\partial C}{\partial q}(q^*, p^*) - p_o^*$$

Q.E.D.

The theorem calls for several remarks :

a) Beyond the use of different techniques, the same ideas underly the proof of theorem 1 and the more intuitive analysis of the preceding subsection : Assumption 6 together with $\frac{\partial \tilde{\mathcal{P}}_o}{\partial t_o} \neq -1$ assures that through the use of lump sum transfers consumers' demands can be induced to move in any direction. Assumption 5 plays the same role for feasible directions of supply changes for all goods but good 0 . Finally the assumption $y_o \in \Lambda_o$ is here replaced by the existence of $\tilde{\mathcal{P}}_o$, single valued and continuously differentiable.

b) The assumption $\frac{\partial\tilde{\mathcal{P}}_0}{\partial t_0}(p^*,T^*,R^*) \neq -1$ means that locally the Government has the power to influence the consumption price π_0 ; if $\frac{\partial\tilde{\mathcal{P}}_0}{\partial t_0}(p^*,T^*,R^*) = -1$ any move of t_0 is counteracted by an opposite move of p_0 .
This assumption is sufficient to exclude some types of behaviors which would here result in inefficient allocations whatever the economic policy, *even in the regular case*. Let us construct an example of such a behavior. Consider a model in which the price maker desires to achieve an a priori given level $\bar{q}$ of commodity zero, whatever the environment. Suppose that there exists $\pi_0(p,t,R)$ such that for any (p,t,R) :

$$\bar{q} = \xi_0(\pi_0(p,t,R),p,t,R)$$

The price making behavior is characterized by :

$$p_0 = \pi_0(p,t,R) - t_0$$

Here $\frac{\partial\tilde{\mathcal{P}}_0}{\partial t_0} \equiv -1$; such a case is excluded by our assumption (which is clearly not a necessary condition).
Observe finally that in the case of the monopolist mentioned above, this assumption reduces to $\frac{\partial \xi_0}{\partial \pi_0} \neq 0$. It is always satisfied because of the monopolist's behavior who maximizes profits.

c) The first part of the analysis rests on the description of feasible states of the economy associated with (2.1) - (2.5) . Program (I) does not make any assumption on the reaction function $\mathcal{P}_0$ beyond the assumption that *the demand for commodity 0 is satisfied by the price maker*. This is the case if the price maker communicates his reaction function, whatever it may be, to a "referee" and in counterpart is constrained to satisfy the demand he is faced with.
It is also the case if the state which finally occurs fulfills

the price maker's expectation, i.e. if the reaction function is "true" at the optimum

$$p_o^* \in \mathcal{P}_o(p^*, T^*, R^*) \quad , \quad q^* \in Q(p^*, T^*, R^*)$$

In general, it is so if the reaction function is derived from the knowledge of the true demand function (see also the concluding comments).

II.4 - MORE ON THE MONOPOLIST CASE.

We will now focus the attention on the case just mentioned of a single monopolist who knows the true demand function.

THEOREM 2.

Let us consider the same optimum and the same assumptions as in theorem 1. Then, the appropriate *rate of subsidy* on the monopolized commodity which restores Pareto efficiency, equals the inverse of its demand elasticity.

PROOF.

The first order condition associated with the monopoly problem is (see section 2)[(3)] :

$$(2.24) \qquad \xi_o(\Pi^*, R^*) + p_o^* \frac{\partial \xi_o}{\partial p_o}(\Pi^*, R^*) - \frac{\partial C}{\partial q}(q^*, p^*) \frac{\partial \xi_o}{\partial p_o}(\Pi^*, R^*) = 0$$

Since $\dfrac{\partial \xi_o}{\partial p_o} = 0$ is excluded we have :

$$(2.25) \qquad [p_o^* - \frac{\partial C}{\partial q}(q^*, p^*)] = - \xi_o^* \left(\frac{\partial \xi_o}{\partial p_o}\right)^{*-1}$$

Reminding that the price elasticity of the demand for commodity zero is

$$e_o = - \frac{p_o}{\xi_o} \frac{\partial \xi_o}{\partial p_o}$$

and using (2.23), we finally obtain : $t_o^* = (e_o^*)^{-1} \quad p_o^*$.

Hence the result if we define the rate of tax as $\frac{t_o^*}{p_o^*}$.

Q.E.D.

If the good is normal the tax is negative ; it is a subsidy. If the good is not normal, the tax is positive. However, in that case it might be in the interest of the monopolist to quote a high price and satisfy only part of the demand. To stick to our model he should then be constrained to fulfill all the demand.

Corollary 1 :

Under the assumptions of theorem 1, if taxing the good sold by the price maker is not allowed, appropriate taxes on the other goods restore Pareto efficiency. In the case of the monopolist, these taxes are given by :

$$t^* = \frac{(e_o^*)^{-1}}{1 - (e_o^*)^{-1}} p^* \quad .$$

PROOF.

The proof goes through to the last paragraph as in Theorem1, where a different normalization of prices is choosen, namely $\pi_o = p_o$

From (2.20), (2.16)

$$t_\ell^* = k'' p_\ell^* \qquad \ell = 1,\ldots,L$$

with $\quad k'' = \dfrac{k - k'}{k}$.

From (2.20), (2.16), (2.8) we have :

$$(k'' + 1) \sum_{\ell=1}^{L} p_\ell^* \frac{\partial x_\ell}{\partial q} = p_o^* .$$

Using (2.25) we obtain :

$$(k'' + 1) \left[p_o^* + \xi_o^* \left(\frac{\partial \xi_o}{\partial p_o} \right)^{*-1} \right] = p_o^*$$

or

$$k'' = \frac{- \xi_o^* \left(\frac{\partial \xi_o}{\partial p_o} \right)^{*-1}}{p_o^* + \xi_o^* \left(\frac{\partial \xi_o}{\partial p_o} \right)^{*-1}} = \frac{(e_o^*)^{-1}}{1 - (e_o^*)^{-1}}$$

Q.E.D.

Theorems 1, 2 generalize immadiately to the case of several price makers, each one choosing the prices of his own subset of monopolized goods. However, corollary 1 clearly does not generalize.

In the case of a price maker producing for intermediary as well as final demand, it is left to the reader to verify that if the monopolized commodity is sold at consumer prices to competitive firms, the same results as above obtain, where ξ_o is now the demand for good 0 by consumers and competitive firms. However, if on the contrary, it is sold at producer prices, Pareto efficiency cannot be usually restored by a tax-system.

Comming back to the case of the monopolist producing for final demand, it is clear that if there is a single good (such as leisure) which cannot be taxed, the same results as in Theorems 1, 2 still obtain since we have one degree of freedom in the normalization of prices. If at the same time, a good, say good 1, cannot be taxed $t_1 = 0$ and the monopolized good cannot be subsidised $t_0 \geq 0$, Pareto optimality cannot be restored in general.

We already discussed the fact that our formulation of the price making behavior was consistent with the existence of several firms deciding jointly on the price to be quoted. However, Theorem 1 assumes that there is only a single price maker producing efficiently : the reader will notice that this assumption is used for obtaining formula (2.22). When there are several price makers two cases have to be distinguished :

- In the first case, the supply of the group of price makers is efficient (relatively to the total production set of this group) : This is the case for example when the price maker sector consists of identical firms —i.e. firms which identical production sets— forming an oligopoly "à la Cournot" and when only the symmetrical Nash equilibria —i.e. those corresponding to equal shares of the total production for all firms— are considered (see Ruffin (1971)). In this case, Theorem 1 remains true and Pareto optimality can be restored ; the set of price makers can be identified with a single fictitious price maker whose production set is the sum of the individual production sets and the argument of Theorem 1 applies.

- In the second case, the supply of the group is not efficient ; since policy tools do not discriminate among oligopolists, first best Pareto optimality cannot in general be restored. In addition, the Kuhn-Tucker multipliers of the associated program which are the shadow prices to be given to public firms for the production of commodities $1,\ldots,L$, are neither proportional to production prices nor proportional to consumer prices.

II.5 - WHAT GOES WRONG IN THE NON REGULAR CASE ?

Consider first the case of a monopolist. Let $\zeta^* = (p_o^*, p^*, T^*, R^*)$ be a maximum of program (I) where $\mathcal{P}_o$ is not single valued. This means that at $e^* = p^*, T^*, R^*$, the maximization of profit by the monopolist yields several maxima among which p_o^* (say 2 and let us call them $p_o^{1*} = p_o^*$ and p_o^{2*}).

We also suppose that in a neighbourhood of e^*, there are around p_o^{1*}, p_o^{2*}, two continuously differentiable branches denoted $\mathcal{P}_o^1$ and $\mathcal{P}_o^2$ on which the necessary conditions for maxima of the monopolist's profit are satisfied.

ζ^* is necessarily an optimum of the following program :

(II) $\quad \text{Max} \sum_{i \in I} \alpha_i V_i(\Pi, R_i)$

(2.26) $\quad \eta_\ell(p) + y_{g\ell} - \xi_\ell(\Pi, R) - x_\ell(p,q) \geq 0 \qquad \ell = 1,\dots,L$

(2.27) $\quad q - \xi_o(p_o^1 + t_o, \Pi, R) = 0$

(2.28) $\quad p_o^1 = \mathcal{P}_o^1(p,T,R)$

(2.29) $\quad p_o^2 = \mathcal{P}_o^2(p,T,R)$

(2.30) $\quad \Psi(p_o^1, p, T, R) \geq \Psi(p_o^2, p, T, R)$

(2.31) $\quad f(y_g) \geq 0$

(2.32) $\quad p \geq 0 \ , \ \pi \geq 0$

Let $(\rho_\ell), \lambda_o, \lambda_1, \lambda_2, \tau, \mu$ be the multipliers associated with (2.26) to (2.31) respectively.

All the functions written in this program are now supposed to be differentiable, and the standard Kuhn and Tucker approach can be used.

Comparing with subsection II.3, one sees that the presence of the (generally) non zero multiplier τ associated with the constraint (2.30) prevents λ^* , the multiplier associated with the constraint (2.28) to be zero. This expresses here the fact already noticed that the production of the monopolist is not locally controllable. Consequently none of the results emphasised in section II.3 obtain. In particular, it must be emphasized that the shadow price to be given to *public firms* are neither *the production prices* (hence production efficiency does no longer necessarily holds) nor the *consumption prices*.

In view of Guesnerie (1972), where deviant price takers —whose supply functions depend only on production prices— are studied, the fact that production efficiency does not hold here is not surprising. Monopolists when they are not fully uncontrollable appear as generalized deviant agents. Production prices have two functions, allocation of production and control of deviants. Consequently it may be desirable in order to perform better the second function, to have production prices different from the shadow prices given to public firms.

However, in Guesnerie (1975), it remains desirable, even in genuine second best situations, to have the shadow prices given to public firms equal to the *consumption prices*. Intuitively, this difference can be seen as follows. Here, the "imperfect" controllability of the price maker can be improved by eventually introducing distortions within the consumers-public production sector, since, through the demand function, the objective function of the price-maker depends on incomes and consumption prices. On the contrary, in Guesnerie (1972) nothing can be gained from such distortions since the objective function of the price-taker depends only on the production prices which can be controlled independently of the instruments used to control the consumers-public production sector.

Let us finally notice that the reasoning presented here from program (II) applies to any price maker which has a differentiable objective function.

III - TAXING PRICE MAKERS WHEN LUMP SUM TRANSFERS ARE AVAILABLE : THE DECENTRALIZATION POINT OF VIEW.

III.1 - DECENTRALIZATION OF FIRST BEST OPTIMA IN AN ECONOMY WITH MONOPOLY : THE PROBLEM.

A superficial view of the preceding results could lead to believe that in a given economy, the presence of monopolies does not prevent from attaining almost all first best optima. Such an assertion could be defended on the following grounds : the situations where the monopoly's optimal decisions are multivalued —the fact that decisions are single valued and continuous is the main requirement for proving theorem I, differentiability being only a technical matter— are in some sense negligible ; for most demand functions ξ_0 , case 2 of section 1 occurs only for values of (p,T,R) which are of Lebesgue measure zero. Hence, theorem 1 would apply almost always. Such a conclusion which is more or less implicitly supported in the literature is not correct. The first part of the argument is right but it does not imply the secont part.

In order to clarify this important point, we shall take a complementary point of view, the decentralization point of view. Considering in a given economy a given first best Pareto optimal state, we ask : is it possible, with only linear taxes, to decentralize this Pareto optimal state when commodity 0 (which is a final commodity) is produced by a monopoly ?

Let ζ be a first best Pareto optimal state of the economy defined in section 2 with a strictly positive production of commodity zero, and let $(x_i(\zeta)),y_g(\zeta),(y_j(\zeta)),y_0(\zeta)$ be respectivly the consumption vectors of consumer i , $i = 1,\dots,I$, and the production vectors of the Government sector, the competitive sector and the firm producing commodity 0 . Let us assume that this Pareto optimum can be decentralized with a unique price system $P(\zeta) = (p_o(\zeta), p(\zeta))$, and consequently with a unique vector of incomes $R_1(\zeta), \dots, R_I(\zeta)$, such that $x_i(\zeta) = \xi_i(P(\zeta), R_i(\zeta))$, $y_g(\zeta) = \eta_g(p(\zeta))$, $y_j(\zeta) = \eta_j(p(\zeta))$, $y_o(\zeta) = \eta_o(P(\zeta))$ (5).

If all firms, except the firm producing commodity zero, are competitive, is it possible to decentralize this Pareto optimum through an appropriate taxation of the monopolized commodity ?

Let us consider the profit function defined in section 1 :

$$\Phi(q,p,t,t_o,R) = [\pi_o(q,\pi,R) - t_o]\, q - C(q,p)$$

Since Pareto optimality requires the proportionality of consumer and producer prices π_ℓ , p_ℓ for $\ell = 1,\dots,L$, it is enough to consider the taxation programs which tax only commodity 0 . Clearly, a necessary and sufficient condition for ζ to be decentralized, despite the presence of the monopoly, is that there exists t_o^* such that $\Phi(q,p(\zeta),0,t_o^*,R(\zeta))$ reaches a positive maximum at $q = q_o(\zeta)$. If we call $Q_o(\zeta,t_o)$ the profit maximizing production levels of the monopoly when the tax is t_o and with demand and price conditions associated with ζ , ζ is decentralizable when $q_o(\zeta)$ belongs to the range of the correspondence $Q_o(\zeta,t_o)$ (6).

The possible non decentralization of $q_o(\zeta)$ seems intuitively to be linked to the non concavity, with the possibility of jumping from one level of production to another level when t_o varies, never choosing intermediary values (see III.2). From this point of view, the possibility of being unable to decentralize a given Pareto optimum does not appear any longer to be pathological (see III.3).

III.2 - SUFFICIENT CONDITIONS FOR DECENTRALIZATION.

In the next theorem, we give conditions on the characteristics of the given Pareto optimum which make it decentralizable.

THEOREM 3.

Let ζ be a Pareto optimum for which the production $q^* = q_0(\zeta)$ of a good 0 is positive and such that :

a) The profit function $\Phi(q,\zeta,t_0)$ exists, is strictly concave and differentiable with respect to $q \in]0,\infty[$ (7) ,

b) $\frac{\partial C}{\partial q}(q^*,p(\zeta))\, q^* - C(q^*,p(\zeta)) \quad - \quad \frac{\partial \pi_0}{\partial q}(q^*,\pi(\zeta),R(\zeta)) \cdot q^{*2} \geq 0$

Then ζ is decentralizable.

When there are several decentralization prices, this result must hold for one production correspondence constructed from one decentralization price.

PROOF.

Given a) a necessary and sufficient condition for q^* to be chosen by the monopolist is :

$$\Phi(q^*,\zeta,t_0) \geq 0$$

$$\frac{\partial \Phi}{\partial q}(q^*,\zeta,t_0) = 0$$

But $\frac{\partial \Phi}{\partial q}(q^*,\zeta,t_0) = \frac{\partial \pi_0}{\partial q}(q^*,\pi,R)\, q^* + \pi_0(q^*,\pi,R) - t_0 - \frac{\partial C}{\partial q}(q^*,p) = 0$

where π,p,R are $\pi(\zeta)$, $p(\zeta)$, $R(\zeta)$.

Let us choose :

$$t_0^* = \frac{\partial \pi_0}{\partial q}(q^*,\pi,R)\, q^* + \pi_0(q^*,\pi,R) - \frac{\partial C}{\partial q}(q^*,p)$$

./.

This t_o^* is admissible if moreover $\Phi(q^*,\zeta,t_o^*) \geq 0$, i.e. $[\pi_o(q^*,\pi,R) - t_o^*]\, q^* - C(q^*,p) \geq 0$ or

$$\frac{\partial C}{\partial q}(q^*,p)\, q^* - C(q^*,p) \quad - \quad \frac{\partial \pi_o}{\partial q}(q^*,\pi,R)\, q^{*2} \quad \geq \quad 0 \qquad (8)$$

Q.E.D.

Remark :

The condition b) is particularly simple if the production set of the monopolist is convex. Then,

$$\frac{\partial C}{\partial q}(q^*,p)\, q^* - C(q^*,p) \quad \geq \quad 0 \quad ,$$

so that a *sufficient* condition for decentralization is (besides a) :

$$\frac{\partial \pi_o}{\partial q}(q^*,\pi,R) \quad \leq \quad 0$$

i.e. that the monopolized commodity be a *normal* good at the given Pareto optimum.

III.3 - EXAMPLES OF NON DECENTRALIZABLE PARETO OPTIMA.

1) An example of a unique Pareto optimum which is not decentralizable.

Let us consider a positive number q^* and a function $\beta_o(q)$ (which is to be interpreted as the inverse demand function) with the following property :

For $\quad q_1 = 3\, q^* \quad , \quad \beta_o(q_1) = \frac{1}{2}\, \beta_o(q^*)$

$\quad q_2 = \frac{1}{10}\, q^* \quad , \quad \beta_o(q_2) = 8\, \beta_o(q^*)$

Observe that :

$$[\frac{1}{2}\,\beta_o(q^*) - t_o]\;3\,q^* - 3\,q^* > [\beta_o(q^*) - t_o]\;q^* - q^*$$

is equivalent to :

$$\beta_o(q^*) \;>\; 4\,t_o + 4$$

$$[8\,\beta_o(q^*) - t_o]\,\frac{q^*}{10} - \frac{q^*}{10} \;>\; [\beta_o(q^*) - t_o]\;q^* - q^*$$

is equivalent to :

$$\beta_o(q^*) \;<\; 4{,}5\,t_o + 4{,}5$$

In view of these inequalities, whatever t_o, a monopoly faced with such an inverse demand function $\beta_o(q)$ and having a linear cost function, will prefer either q_1 or q_2 (or both) to q^* . Consequently, he will never choose q^* .

Now, let us consider the following simple economy : there are two commodities labeled 0 and 1 ; 0 is a consumption good, 1 is labor. There is one firm producing commodity 0 from commodity 1 , with constant returns to scale (the transformation curve is $y_o + y_1 = 0$).

There is a single consumer with preferences such that the only optimum of the economy corresponds to the production plan $y_o^* = 1$, $y_1^* = -1$; the demand for commodity 0 when the consumer is faced with the price system [.5,1] is 3 and the demand for commodity 0 when the consumer is faced with the price system [8,1] is $\frac{1}{10}$.

(In Figure 2, a family of indifference curves meeting these conditions is constructed).

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In such an economy, if the firm is a monopoly, there exists no tax scheme which enables to decentralize the unique optimum, since from the discussion above the producer will never want to produce the plan $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

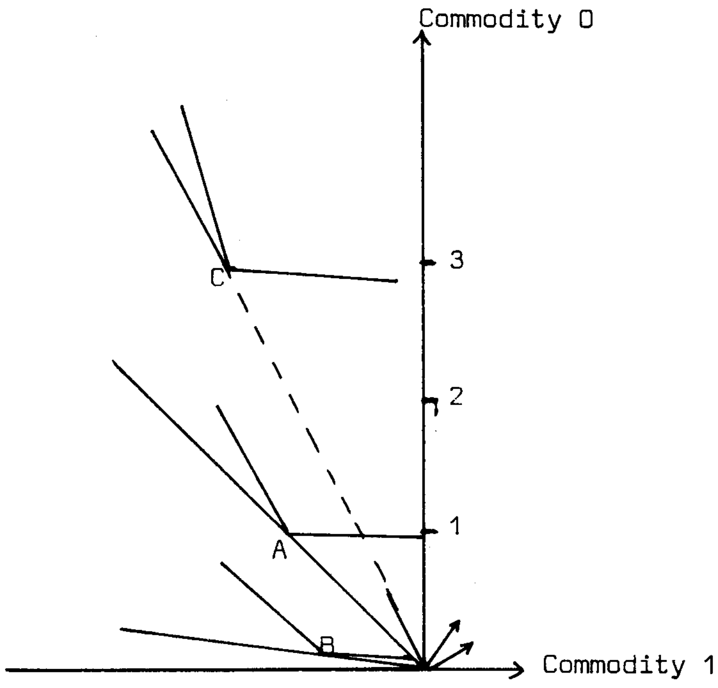

FIGURE 2.

The indifference curves are constructed so that :

. The slopes of the right hand side of the indifference curves passing through A , B , C are identical and take a value between $-\frac{1}{10}$ and 0 ;

. the left sides of the indifference curves passing through B , A , C have slopes which are respectively between $-\frac{1}{10}$ and -1, -1 and -2 , -2 and $-\infty$.

These indifference curves can be appropriately completed to represent a continuous quasi concave utility function. Also, a differentiable quasi concave utility function could easily be constructed to give A , B , C as tangency points.

2) <u>Example where no Pareto optimum is decentralizable</u>.

Consider an economy with I consumers, two commodities (commodity 0 and commodity 1) and one firm. All consumers have the same utility function :

$$\begin{aligned} u(x_o,x_1) &= x_o\, x_1 \qquad \text{if} \quad x_o \geq \frac{x_1}{2} \\ &= x_o \qquad\quad \text{if} \quad x_o < \frac{x_1}{2} \end{aligned}$$

The endowment of the economy in commodity 1 is 1 and the technology is such that $y_o + y_1 = 0$.

Let $(\pi,1)$ be the normalized consumption price vector. Good 0 is sold by a monopolist.

In this economy, all Pareto optima are associated with a production of commodity 0 at the level $\frac{1}{2}$. The decentralization price vector is (1,1) so that total associated income is always 1.

Let $(R_1,\dots,R_I)$ be the vector of income in the economy. The demand functions for commodity 0 are easily derived as :

$$\begin{aligned} \xi_{oi}(\pi,1) &= \frac{R_i}{2\,\pi} \qquad \text{if} \quad \pi < 2 \\ &= \frac{R_i}{2+\pi} \qquad \text{if} \quad \pi \geq 2 \;, \quad i = 1,\dots,I \end{aligned}$$

At any Pareto optimum, $\sum_{i=1}^{I} R_i = R = 1$ and the profit function of the monopolist can then be written as :

$$\begin{aligned} \Psi &= \frac{1}{2} - (t_o + 1)\, q \qquad \text{if} \quad \pi < 2 \quad \text{or} \quad q > \frac{1}{4} \\ &= 1 - (t_o + 3)\, q \qquad \text{if} \quad \pi \geq 2 \quad \text{or} \quad q \leq \frac{1}{4} \end{aligned}$$

This profit function is described on Figure 3.

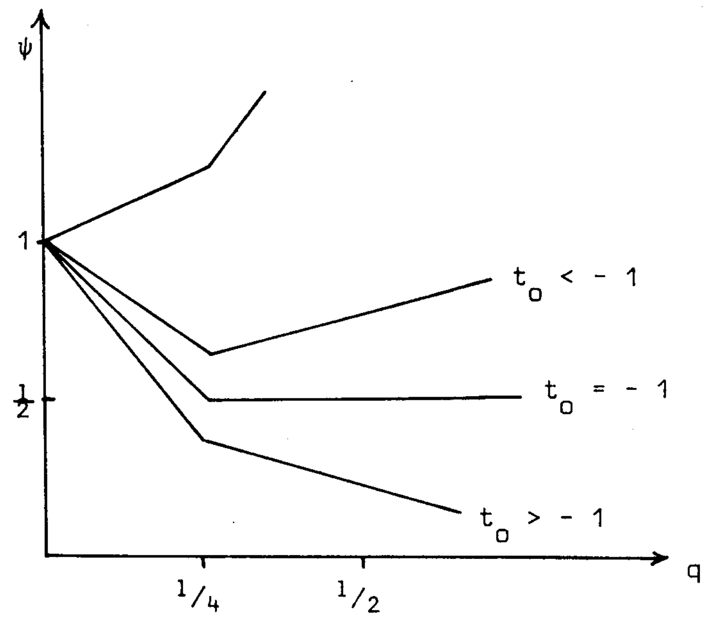

FIGURE 3.

From, the profit function pictured in Figure 3 , it is clear that there exists no tax system such that the monopoly agrees to produce $q = \frac{1}{2}$. Hence, the result.

Note that, with a lower bound ε on the production set of the firm, there would exist in this economy a monopolistic equilibrium at $q^* = \varepsilon$ for any tax $t_o > -3$.

Let us comment somewhat on this example. If there is no subsidy or if the subsidy is too small, the monopolist wants to produce a minimal amount to exploit the fact that commodity zero is a necessary good. The Center then tries to have the monopolist produce more by increasing the subsidy (here without success) and at some level of subsidy the monopolist would like to produce an infinite amount.

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In designing the optimal tax there is clearly a tendency to stop the increase of subsidy at a point where further increase would involve a jump in behavior. The monopolist does not produce enough ; one attempts to have him produce more and stop the level of subsidy at a point at which he is ready to jump to an excessive production.

Hence, in this case, the optimal taxation puts always the monopolist at a "catastrophic" level of production (the catastrophy is not due to special values of exogeneous variables, but is the result of the optimal choice of the control variables). The study of the generality of this result would certainly deserve further work.

3) First conclusions in the case where lump sum transfers are available.

. In the "good" regular case, first best Pareto optimality can be restored with linear taxes. In the "bad" case (not regular), maximization of a social welfare function leads to a genuire second best. Furthermore, the production of the price maker in such a state is highly unstable with respect to small changes of the environment or of the control variables. No simple results concerning optimal taxation and optimal public production can be exhibited.

. When the profit function of the monopolist is not concave, the "bad" case cannot be viewed as pathological. It will be often the case that by varying the weights of the social welfare function, bad case and good case will successively appear i.e. from the decantralization point of view that a subset of first best Pareto optima will not be reachable.

These conclusions can be illustrated through the following diagram in a 2 consumers, 3 goods economy.

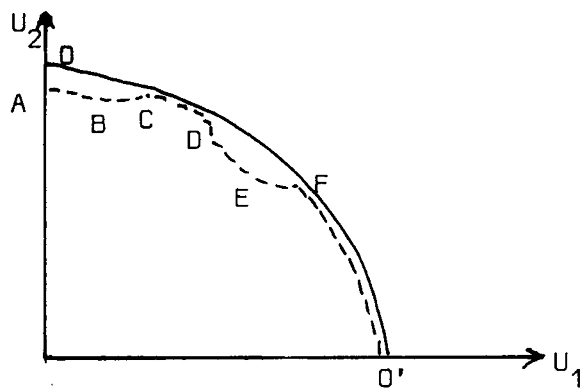

FIGURE 4.

. In figure 4, we represent in the space of utilities on the one hand the attainable levels of utilities in the physical model, the upper frontier of which is the solid curve OO' (corresponding then to first best Pareto optima) ;
on the other hand, the attainable levels of utilities when linear taxes can be used in an economy with a price maker, the upper frontier of which is the doted curve ABCDEFO' .
The solid curve and the dotted curve coïncide on CD and FO' :
the corresponding first best optima are decentralizable in the model with a price maker. The solid curve and the dotted curve differ on ABC , DEF . Social welfare functions leading to such optima in the utility space, give second best situations.

. Figure 5 visualizes on the $\mathbb{R}^2$ production frontier of the monopolist the paths of the monopolist's optimal production plan associated with the optima ABCDEFO' of figure 4..
The solid line OO' associated with OO' in figure 4 represents production plans associated with first best allocations : under standard assumptions it is a continuous line.
On the contrary the dotted line ABCDEO' corresponding to the monopolist's optimal production plans in the model with linear taxes is discontinuous :

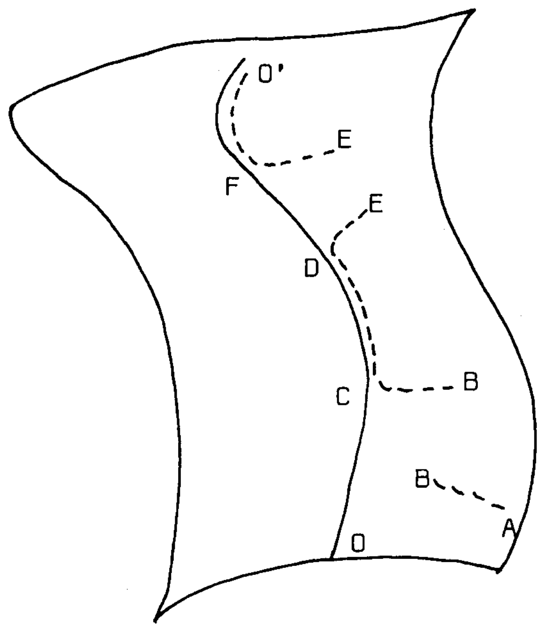

FIGURE 5.

on the figure it jumps at B and E which correspond either to discontinuities in the control variables or in the responses of the monopolist.
This dotted line coïncide with OO' on CD and FO' where the monopoly response is stable, while it is unstable on ABC , DEF .

Let us finally notice that the topological properties of the line ABCDEFO' suggested in figure 5 have to be considered as illustrative. A more precise study of discontinuity points would be needed.

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IV - TAXING PRICE MAKERS : THE INTERACTION OF ALLOCATIONAL AND DISTRIBUTIONAL ISSUES.

The developments of the preceding sections rested on the assumption that lump sum transfers were feasible. This assumption, which allowed us to put aside distributional issues ans to concentrate on the corrective role of taxation in the allocation of resources, is now relaxed.

IV.1 - A "DIAMOND-MIRRLEES" ECONOMY.

We first consider an economy "à la Diamond-Mirrlees" (1971) in which there is a price maker producing for final demand.

The feasible states of the economy are defined by the following system of inequalities :

$$(4.1) \qquad \eta_\ell(p) + y_{g\ell} - \xi_\ell(\Pi,\vec{0}) - x_\ell(p,q) \geq 0 \qquad \ell = 1,\ldots,L$$

$$(4.2) \qquad q - \xi_o(\Pi,\vec{0}) = 0$$

$$(4.3) \qquad p_o \in \mathcal{P}_o(p,T,\vec{0})$$

$$(4.4) \qquad f(y_g) \geq 0$$

$$(4.5) \qquad p \geq 0 \ , \ \pi \geq 0$$

where $\vec{0}$ is the vector $(0,\ldots,0)$ of non labor incomes.

A system of indirect linear taxes enables the Government to disconnect production and consumption price systems. Profits of competitive firms and of the price maker are 100 % taxed so that consumers have no other income than their labor income, i.e., $R_i = 0$, $i = 1,\ldots,I$.

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This latter hypothesis rules out part of the distributional problems created by the existence of a monopoly which are related with the distribution of its profits to shareholders. Therefore, the model of this section must be considered a step in the process of understanding the more realistic but more complicated model of subsection IV.2, IV.3 in which profits are distributed.

Let us consider the two following maximization programs :

(P) $$\text{Max} \sum_i \alpha_i \, V_i(\Pi, 0)$$

under (4.1) to (4.5)

(P') $$\text{Max} \sum_i \alpha_i \, V_i(\Pi, 0)$$

under (4.1), (4.4), (4.5) and

(4.6) $$\xi_0(\Pi, 0) \leqq \eta_0(P)$$

where $\eta_0(P)$ is the competitive supply of commodity 0 when the production price system is P, i.e., the supply of a competitive firm which would have the same production possibilities as the price maker.

(P) is the program to consider in an economy "à la Diamond-Mirrlees" when commodity 0 is produced by a price maker and (P') is the analogous program when the supply of commodity 0 is competitive.

A partial characterization of the maxima of program (P) is given by :

<u>THEOREM 5.</u>

Let (P^*, T^*, y_g^*) be an optimal solution of program (P) with $P^* > 0$. Under A1 to A4 , and under regularity conditions on demand (of the Diamond-Mirrlees type), if in a neighbourhood of that optimal solution the correspondence $\mathcal{P}_o$ reduces to a single valued and continuously differentiable function such that $\frac{\partial \mathcal{P}_o}{\partial t_o}(p^*, T^*, R^*) \neq -1$, and if A5 is satisfied at the optimum, then :

a) The shadow prices associated with the optimal production plans of the public firm coïncide with the production prices

b) The optimum of program (P) satisfies the Kuhn and Tucker conditions of program (P').

PROOF.

It is easy to see that the necessary conditions associated with program (P) are formally identical to conditions (2.6) to (2.11) of section II.3. $\mu^* > 0$ is here assured by the Diamond-Mirrlees demand conditions. It follows that the dual variable λ^* associated with constraint (4.3) equals zero. The argument leading to formula (2.16) can be repeated. Hence :

$$\rho_\ell^* = k\, p_\ell^* \quad , \qquad \ell = 1,\dots,L$$

From (2.8) , we have :

$$\rho_o^* = k\, \tilde{\rho}_o \quad ,$$

where

$$\tilde{\rho}_o = \sum_{\ell=1}^{L} p_\ell^* \frac{\partial x_\ell}{\partial q}$$

is the marginal cost of producing commodity zero.

Let $(\rho_o', \dots, \rho_L')$ be a vector of dual variables associated with an optimal solution of program (P') . It is immediate to see that the optimal solution of program (P) satisfies the Kuhn and Tucker conditions of (P') by choosing $\rho_\ell' = \rho_\ell^*$, $\ell = 0,\dots,L$, as defined above.

Q.E.D.

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Theorem 5 states that at an optimum where the response function of the price maker is differentiable, production must be efficient (a) and the optimum is a local maximum "à la Diamond-Mirrlees" (b) .

The optimal tax on commodity 0 , $t_0^* = \pi_0^* - \rho_0^*$, equals now the sum of a "corrective" tax $\tilde{p}_0 - p_0^*$ and a distribution tax $(\pi_0^* - \tilde{p}_0)$ "à la Diamond-Mirrlees" on which nothing general and independant of the normalization rule can be said.

If the differentiability assumption on $\tilde{\mathcal{P}}_0$ does not hold at the optimum of the program, the necessary conditions must be derived along the lines investigated in section II.5. The reader can easily see that, generally in this case, production efficiency is no longer desirable, the response of the price maker is unstable with respect to the value of policy tools and the tax formula are no more of the same form as the tax formula "à la Diamond-Mirrlees" obtained in theorem 5.

There are no grounds for asserting that an optimum of program (P) in theorem 5 is a *global* optimum of program (P') . It was only proved that it satisfies the Kuhn and Tucker conditions of program (P') . Contrary to what happened in section II , necessary conditions are not sufficient for obtaining a global optimum.

Given a *second best* Pareto optimum solution of program (P') one may ask, as in section III , if it can be decentralized when firm 0 is a monopoly. A sufficient condition for decentralization similar to the one derived in theorem 3 , is easily obtained. However, the similarity with sections II, III should make clearly non pathological the fact that some second best Pareto optima corresponding to the Diamond-Mirrlees assumptions are not decentralizable when commodity zero is produced by a monopoly.

The reader will find useful to illustrate these conclusions with diagrams similar to the ones presented in section III.

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IV.2 - DISTRIBUTION OF PROFITS WITH A COMPLETE SYSTEM OF TAXES.

The next step is to give up the assumption that profits of the price maker are taxed 100 % . We consider now the case where the profit of the monopoly, B , is distributed to consumers according to fixed shares $\theta_i (\sum_{i=1}^{I} \theta_i = 1)$ and where consumption prices and production prices can still be disconnected with a system of linear indirect taxes.

A feasible state of the economy is then defined by a vector (Π, P, R, B, q, y_g) which satisfies

$$(4.7) \qquad \eta_\ell(p) + y_{g\ell} - \xi_\ell(\Pi,R) - x_\ell(p,q) \geq 0 \qquad \ell = 1,\ldots,L$$

$$(4.8) \qquad \xi_o(\Pi,R) - q = 0$$

$$(4.9) \qquad p_o \in \mathcal{P}_o(p,T,R)$$

$$(4.10) \qquad f(y_g) \geq 0$$

$$(4.11) \qquad B - p_o\, \xi_o(\Pi,R) + C(q,p) = 0$$

$$(4.12) \qquad R_i = \theta_i\, B \qquad i = 1,\ldots,I$$

$$(4.13) \qquad B > 0 \ , \quad p_o \geq 0 \ , \quad p \geq 0 \qquad i = 1,\ldots,I$$

Consider a feasible state of (4.7) to (4.13) ; the price quoted by the monopoly satisfies (see section II.4)

$$(4.14) \qquad 1 = \frac{1}{e_o} + \frac{\frac{\partial C}{\partial q}}{p_o}$$

where e_o is the price elasticity of the demand for commodity zero and the profit of the monopoly is $B = p_o\, \xi_o(\Pi,R) - C(q,p)$. To the same feasible state we can associate a different normalization of prices, for example, multiply p by λ . From (4.14) we know that p_o is then multiplied also by λ . The associated profit of the firm is now $B' = \lambda B$

which can be made as desired by choosing an appropriate λ .

Consequently, the constraint (4.11) can be ignored, B is now a decision variable and (4.12) is equivalent to having positive lump sum transfers in fixed proportions. In the differentiable case, the first order conditions of the above maximization problem are closely similar to those obtained by Diamond (1975) in his model with a poll tax. In particular, production efficiency holds by the monotonicity of preferences and a tax formula of the many persons Ramsey type is obtained.

However, this first approach to the distributional questions raised by the distribution of the monopolist's profits is quite unsatisfactory ; it leads to give a crucial role to the highly irrealistic assumption of an independent choice of normalization for consumer and producer prices.

The case $B = 0$ has been excluded in the above analysis because it is a non differentiable case : if we consider a non zero production level with such a profit, we know that there exists another of the profit function for the zero production level.

This brief analysis shows that if one wants to capture the purely redistributive problems raised by the existence of monopolies, the complete disconnection of production and consumption prices must be given up.

IV.3 - DISTRIBUTION OF PROFITS WITHOUT A COMPLETE DISCONNECTION OF CONSUMPTION AND PRODUCTION PRICES.

In the above subsection although profits were not explicitly taxable, they could be controlled through an appropriate normalization of production prices. To rule out this phenomenon, we introduce a new rigidity by assuming that commodity one is not taxed, i.e., $t_1 = 0$.

a) Profits are taxable.

Let ε be the rate of profit taxation, $\varepsilon \in [0,1]$.

The feasible states of the model are now defined by

$$(4.15) \qquad \eta_\ell(p) + y_{g\ell} - \xi_\ell(\Pi,R) - x_\ell(p,q) \leq 0 \qquad \ell = 1,\dots,L$$

$$(4.16) \qquad q - \xi_o(\Pi,R) = 0$$

$$(4.17) \qquad p_o \in \mathcal{P}_o(p,T,R)$$

$$(4.18) \qquad f(y_g) \geq 0$$

$$(4.19) \qquad B = \psi(p_o,p,T,R)$$

$$(4.20) \qquad R_i = \theta_i \,\varepsilon\, B$$

$$(4.21) \qquad t_1 = 0$$

$$(4.22) \qquad p \geq 0 \;,\; \pi \geq 0 \;,\; B \geq 0$$

Let us consider a second best Pareto optimum for which $\mathcal{P}_o$ is differentiable, $\varepsilon \in]0,1[$ and let β , ν_i , $i = 1,\dots,I$ be the Kuhn and Tucker multipliers associated to (4.19) (4.20).

The Kuhn and Tucker conditions associated with B , (ε) , are

$$(4.23) \qquad \beta - \sum_i \nu_i \theta_i \varepsilon = 0$$

$$(4.24) \qquad - \sum_i \nu_i \theta_i B = 0$$

which imply $\beta = 0$.

The other Kuhn and Tucker conditions are then similar to those obtained in the previous section. As in section 2 one obtains $\lambda = 0$, i.e., the only effective constraints in addition to (4.15) (4.16) (4.18) (4.22) are (4.20) (4.21).

To sum up we have :

THEOREM 6.

Let $\zeta^* = (P^*, T^*, y_g^*)$ a second best Pareto optimum solution under the constraints (4.15) to (4.22) . Under A1 - A4 and the differentiability and regularity conditions analogous to those of theorem 5, and if in addition $\varepsilon^* \in]0,1[$, ζ^* satisfies the Kuhn and Tucker conditions associated to the constraints (4.15) (4.16) (4.18) (4.20) (4.21) (4.22) .

In particular the necessary Kuhn and Tucker conditions are the same as those obtained in a program where the Government can fix independently the production level of the monopolist and the level of the monopolist's profit while still being constrained by the cost minimization behavior of the monopolist. Because of (4.21) production efficiency is not generally desirable.

When the differentiability assumption is no longer satisfied, the above results do not hold anymore. In addition, we obtain then the usual "instability" of the monopolist's behavior.

b) Monopoly profits are not taxable.

In this case, the feasible states are defined by (4.15) to (4.22) with $\varepsilon = 1$ in (4.20).

Even in the differentiable case, one does not get $\beta = 0$ and $\lambda = 0$. The two objectives of the Government achieved in a), namely the "control" of the production level of the monopoly and of the level of profits of the monopoly are no longer jointly attainable ; they become conflicting objectives. In the non differentiable case we obtain in addition the instability mentionned above.

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CONCLUDING COMMENTS.

The theory of optimal taxation of a price maker developed in this paper has made a number of very strong informational assumptions concerning the Government and the economic agents that we can now discuss.

Our direction is to assume that the Government and the price makers know the *true* demand functions. This assumption of perfect knowledge of demand conditions has been found irrealistic in the literature because of the complexities of the general equilibrium computations which are implicit in it. However this well taken criticism has led through the use of perceived demand function to an unsound reasoning theory.

Let $\hat{\xi}_0(\Pi,R)$ be the demand function perceived by the price maker[9]. This function which must be true at the equilibrium is therefore indexed by $e^* = (\Pi^*,R^*)$, equilibrium values assumed to be unique and is denoted $\hat{\xi}_0^{e*}(\Pi,R)$. Let us consider the economy of section 4 without taxes and let us assume that there exists a unique equilibrium e^* . Then let us consider a perceived demand function $\hat{\xi}_0^{e*}(\Pi,R)$. In order to define the optimal taxation problem, we must not only assume that the Center knows this *perceived* demand function, but also the way this function changes with the different equilibria associated with different tax schemes. If this perceived function was fixed during a long period, one could argue that by observing the behavior of the monopolist, the Government could discover this function at least locally (even then one would have probably to assume that the production possibilities of the price maker are known to the Center ; otherwise there would be an identification problem). However, if this function is substantially different from the true one it is obvious that there will be some learning over time ; then, it is clearly impossible to assume that the Government knows also the way the price maker learns. The informational issue is clearly at the heart of the matter and a theory of optimal taxation with perceived demand functions must face it.

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Another possibility, as a first step in the study of optimal taxation with price makers, is to assume that the perceived demand functions are close enough to the true ones to be stable. Then, one must face another difficulty, the non concavity of profit functions ; it was essentially the topic of this paper which can be to some extent interpreted with such perceived demand functions instead of true ones.

The second main assumption of our approach has been to consider price makers with a Nash behavior with respect to the whole environment except the price of their own output. It is a strong informational hypothesis and to assume a more strategic behavior would have been unwise in a first step. It is clear that more realistic assumptions can be made, in particular in the case of oligopolists, and this should be the object of further work. Note however, that it is easy to see that if the monopolist takes into account the effect of his price change over the distribution of income through the change in his own profit, the results would be unchanged. The reason is that by the first order condition, the derivative of his profits with respect to his price is zero.

Finally, it is important to observe that we did not touch the question of strategic behavior of the monopolist with respect to the Government. The monopolist knowing that he will be taxed may attempt to distort his behavior in order to conceal his true characteristics and obtain a more favorable tax.

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FOOTNOTES.

(1) The set of feasible states we are considering is such that all markets are cleared through price adjustments. We may also interpret the program as if the policy tools of the Government were p,T and R and if he behaved as an auctioneer.

We assume throughout the paper that the set defined by equations (2.1) to (2.5) is non empty and that program (I) has solutions. We refer to the literature on existence proofs (see Roberts and Sonnenschein (1977)) for sufficient conditions of existence when taxes are zero. Let us note only that the presence of taxes makes the existence question more tractable ; for example, when the commodity produced by the monopoly is not a "necessary" good, the economy can be reduced to an economy without monopoly by using a large enough tax on this commodity. Looking at compactness is also outside the scope of this paper.

(2) Throughout the paper, the regularity conditions of the Kuhn-Tucker theorem are assumed to be valid.

(3) The reaction function of a monopolist around a point where it is univalent is locally defined by the first order condition (2.24).

Moreover, in a neighbourhood of such a point, profit is strictly positive ; $B = 0$ at a positive level of production is excluded by the differentiability assumption since we have in that case at least two optimal policies, another one corresponding to a zero production level.

(4) Since $\frac{\partial C}{\partial q} \geq 0$, $p_o + \xi_o\left(\frac{\partial \xi_o}{\partial p_o}\right)^{-1} \geq 0$ from (2.24) hence $1 - \frac{1}{e_o} \geq 0$, i.e. at the optimal production level, the absolute value of the

elasticity of demand is always larger or equal to one. Therefore $k'' > 0$ (< 0) when the good is normal (not normal), paralleling the comment made or theorem 2.

(5) As η_j denotes the competitive supply function of competitive firm j , η_g , η_o respectively denote the competitive supply functions associated with Y_g and Y_o , the production sets of the public sector and of the monopoly.

(6) Obviously $q_o(\zeta) \in Q(\zeta, t_o)$ is impossible for $t_o < -p_o(\zeta)$.

(7) The profit function is written here as a function of q , t_o and ζ which summarizes the other characteristics of the given Pareto optimum. Its existence relies on the invertibility of $\pi_o(.)$. Observe also that a profit function strictly concave in q for a value of t_o is strictly concave for any t_o .

(8) The same result obtains more generally if $\frac{\partial Q_o}{\partial t_o} \neq 0$ since

$$y_o - \bar{y}_o = \frac{\partial Q_o}{\partial p}(p - \bar{p}) + \frac{\partial Q_o}{\partial t}(t - \bar{t}) + \frac{\partial Q_o}{\partial R}(R - \bar{R}) + \frac{\partial Q_o}{\partial t_o}(t_o - \bar{t}_o) \quad .$$

(9) Negishi (1961) was the first to introduce in a different context the concept of perceived demand function with the only requirement that the function be correct at the equilibrium.

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REFERENCES

ARROW, K. (1971) :
"The Firm in General Equilibrium Theory"
in MARRIS, and A. WOOD (ed.), The Corporate Economy : Growth, Competition and Innovative Potential, London MacMillan

ARROW, K. and HAHN, F. (1971) :
General competitive analysis
San Francisco, Holden-Day

BERGSON, A. (1972) :
"Optimal pricing for a public enterprise"
Quartely Journal of Economics, 136, pp 518-542

BOHM, P. (1967) :
"On the theory of second best"
Review of Economic Studies, 14, pp 301-314

BOITEUX, M. (1956) :
"Sur la gestion des monopoles publics astreints à l'équilibre budgétaire",
Econometrica, 24, pp 22-40

DEBREU, G. (1959) :
Theory of Value.
New York, John Wiley and Sons

DIAMOND, P. (1975) :
"A Ramsey many person tax rule"
Journal of Public Economics, 4, pp 335-342

DIAMOND, P. and J. MIRRLEES (1971) :
"Optimal taxation and public production"
American Economic Review, 61, pp 8-27, pp 261-278

FITZROY, F. (1974) :
"Monopolistic equilibrium, non convexity and inverse demand"
Journal of Economic Theory, 7, pp 1-16

GABZEWICZ, J. and J.P. VIAL (1972) :
"Oligopoly "à la Cournot" in general equilibrium analysis",
Journal of Economic Theory, 4, pp 381-400

GUESNERIE, R.S. (1975) :
"Public production and taxation in a simple second best model" ;
Journal of Economic Theory, 10, pp 127-156

LAFFONT, J.J. and G. LAROQUE (1976) :
"Existence d'un équilibre général de concurrence imparfaite : une introduction",
Econometrica, 44, pp 183-194

Mc FADDEN, D. (1969) :
"A simple remark on the second best Pareto optimality of market equilibria",
Journal of Economic Theory, 1, pp 26-38

MARSCHAK, J and R. SELTEN (1974) :
General equilibrium with price setting firms,
Berlin, Heidelberg, and New York, Springer Verlag

MIRRLEES, J. (1977) :
"The Theory of optimal taxation", September 1977
Nuffield College

NEGISHI, T. (1961) :
"Monopolistic competition and general equilibrium",
Review of Economic Studies, 28, pp 196-201

ROBERTS, J. and H. SONNENSCHEIN (1977) :
"On the foundations of the theory of monopolistic competition",
Econometrica, 45, pp 101-113

RUFFIN (1971) :
"Cournot oligopoly and competitive behaviour",
Review of Economic Studies, 38, pp493-502

SANDMO, A. and J. DREZE (1971) :
"Discount rate for public investments in closed and open economies",
Econometrica, 38, pp 395-412