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ON THE DIRECTION OF TAX REFORM

by

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LA DIRECTION DE LA REFORME FISCALE

L'approche traditionnelle des études d'économie normative concentre l'attention sur la caractérisation de politiques "optimales" (théorie du "second best"). La perspective ainsi adoptée conduit à des résultats qui, dans la mesure où ils ne permettent pas le calcul effectif des dites politiques optimales, ont l'inconvénient d'être peu opératoires. En d'autres termes, il semble plus important concrètement de connaître la direction de la réforme , c'est-à-dire le sens des petites modifications à apporter à la politique économique pour améliorer le système en regard d'un certain critère, que d'avoir des informations partielles et abstraite sur l'agencement "optimal" des mesures de politiques économiques.

Dans cet esprit le papier présente un algorithme de "réforme fiscale" dans un cadre simple.

La première partie du papier définit :

- Le cadre théorique de la réflexion. C'est celui d'un modèle qui se prête bien à l'étude théorique de la fiscalité indirecte et de son impact sur l'allocation des ressources et la distribution des revenus (modèle type Diamond Mirrlees).

- L'état initial du système : C'est un équilibre relativement à un système fiscal donné (système fiscal de type TVA).

- Quelques résultats techniques préliminaires

La seconde partie de la note présente les conclusions obtenues.

- Tout d'abord, on caractérise les modifications de taxes qui sont "réalisables".

- Parmi ces modifications réalisables du système fiscal, on exhibe l'ensemble de celles qui sont satisfaisantes en un sens Parétien. On donne des conditions pour que cet ensemble ne soit pas vide. On obtient

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ainsi comme sous produit de l'analyse la caractérisation des états optimas de second rang à laquelle conduit l'approche traditionnelle de la théorie de la fiscalité optimale . Cependant le résultat ne repose ici ni sur une fonction d'utilité collective ni sur le recours explicite à des techniques d'optimisation.

- On met aussi en évidence qu'un des résultats essentiel de cette théorie -la propriété d'efficience de la production - vrai à l'optimum peut être mis en défaut le long d'un chemin y conduisant. Pour améliorer le système au sens Parétien,il est nécessaire et inévitable, en certains états, d'accepter des inefficacités temporaires.

- La mise en place d'une réforme de grande ampleour requiert la résolution de systèmes d'équations différentielles qui sont présentés, et qui font clairement apparaître les antagonismes et les conflits entre agents dans le processus de réforme.

Despite of the rapid growth and recent achievements of the optimal taxation literature, the methods and approach of this current of research have been criticized on several grounds. Criticisms came either from contributors in this field aware of its shortcomings, —"internal" criticisms— or from specialists reluctant to the whole approach —"external" criticisms—. "Internal" criticisms put the emphasis on the fact that the knowledge of optimal taxes may be useless for practical purposes since "actual changes are slow and piecemeal"(M. FELDSTEIN [1975]) and that "policy changes which appear to be steps in the right direction but stop short of attaining the full optimum can reduce welfare" (DIXIT [1975]). "External" criticisms express skepticism about the use of a social welfare function which does not "exist, independantly of the mutual adjustment process itself" (BUCHANAN [1975]) and correlatively stress out that the optimal taxation framework ignores the existing tax system, the conflicts about changes, and the considerations of horizontal equity which have been an important topic of the previous public finance literature (see MUSGRAVE [1959]). Such objections are clearly exposed in BUCHANAN [1975] who advocates returning to a previous theoretical tradition, that he terms "Wicksellian".

Both "internal" and "external" critiques could to some extent agree with M. FELDSTEIN's proposition [1975] aimed at shifting the emphasis from "tax design" which is the topic of the optimal taxation literature, to "tax reform" which "takes as its starting point, the existing tax system and... consider the position of each individual before as well as after any proposed change".

Actually, as convincingly argued by Martin FELDSTEIN the problem of tax reform has many dimensions that cannot easily be simultaneously captured. The purpose of this paper is to focus the attention on one aspect of studies in the field of "tax reform", concerning what can be termed the problem of the direction of tax reform. More precisely, instead of asking the traditional question "what are the characteristics of optimal taxes ?", one will attempt to shed light on the following problem "given an existing tax system, are there feasible and "satisfactory" (relatively to a given criterion) small moves of the tax system which can be implemented ?" In other words, "are there desirable directions of tax reform ?" .

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Let us note that preoccupations underlying the above questions are not novel and that problems of a similar type have been raised in the literature.(see for example the recent work of DIXIT [1975])and specially in the international trade literature — cf. NEGISHI [1971]—.The specific goal of this note is to consider the above questions in a Diamond-Mirrlees'world, i.e. in the framework of the most well-known model of the optimal taxation literature, with the aim of exploring the relationships between the "tax design" and the "tax reform" points of view alluded to above. Keeping in mind this line of arguments, two kinds of results will be obtained :

- First starting from a given equilibrium corresponding to the existing tax system, one will explore neighbour equilibria in order to exhibit directions of tax reform which are both feasible and satisfactory in a Pareto sense i.e. satisfactory for all agents. When such directions will actually exist, they will be characterized in a way allowing an effective computation of the corresponding change of taxes. A striking result will appear making clear that temporary inefficiencies in the production sector may be desirable in the process of tax reform dispite of the "efficiency" property which holds for optimal tax design.

- Second best optimal states are states for which no feasible and satisfactory changes in taxes exist. So, characteristics of optimal taxes will appear as a joint product of the analysis of the directions of tax reform. This will provide a different proof of some of the Diamond-Mirrlees'results, a proof which does not refer neither to a concept of social welfare function, nor to explicit optimization techniques.

The note will proceed as follows :

In a first paragraph, all elements of the problem — model and notations AI, the starting point in B1— , and some preliminary results — CI— and definitions — DI— will be presented. In paragraph B, feasible moves of taxes -BI- and Pareto improving moves -BII-will be characterized. The results will be commented in III.

A - PRESENTATION OF THE PROBLEM AND PRELIMINARY WORK.

I - MODEL AND NOTATIONS :

One will adopt notations similar to those used by DIAMOND-MIRRLEES in [1971].In the economy, there are :

H households indexed by $h = 1,\ldots,H$

n commodities indexed by $k = 1,\ldots,n$. Commodities are specialized in the sense that commodities 1 to n_1 can only be consumed in negative quantities (or supplied) and commodities $n_1 + 1$ to n_2 can only be consumed in positive quantities (or demanded), this being true for each household.

This can be formalized through the definition of adequate consumption sets X_h . Assumptions on such X_h , that will be made in the following are gathered in H1)

H1) X_h is such that commodities can be partitioned in two specialized sets $(1,\ldots,n_1)$, $(n_1 + 1,\ldots,n)$. Furthermore X_h is convex and bounded below. Each household has preferences on X_h , represented by a utility function u_h which satisfies H2)

H2) u_h is a strictly quasi-concave function, and u_h is monotonic (i.e. $x_h > x'_h \Rightarrow u_h(x_h) > u_h(x'_h)$) and differentiable.

Faced with price system(1) $q \in \mathbb{R}^n_+ - \{0\}$ (q is the consumer price system), household h, which has no other source of income than his labor income, determines his consumption choice, by solving the program :

$$\text{Max } u_h(x_h) \quad \{x_h \in X_h \ , \ q \cdot x_h \leq 0\}$$

H1)-H2) assure that the solution of this program is unique and that at the optimum the budgetary constraint is tight.

Let us call $x_h(q)$ this solution : $x_h(q)$ is the demand vector of household h . $x_h : (\mathbb{R}^n_+ - \{0\}) \to \mathbb{F}^n)$ is the demand function of household h.
It is homogeneous of degree zero and such that $q.x_h(q) = 0$

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(1) In the following, unless explicit contrary statement, price vectors will be line vectors and quantity vectors will be column vectors. A^T will denote the transposed of A (A being a vector or matrix).

$V_h(q) = u_h(x_h(q))$ is the indirect utility function.

The aggregate net demand is $X(q) = \sum_{h=1}^{H} x_h(q)$.

The production sector has production possibilities described through the production function $G(y) \leq 0$.

In the following, assumption H3) will be made.

H3) G is a strictly quasi convex function defined on $\mathbb{R}^n$ and G is monotonic :

$y > y' \implies G(y) > G(y')$

Given a production price system $p \in \mathbb{R}^n_+ - \{0\}$ the competitive supply of the production sector is determined by solving the program :

$$\text{Max } p \cdot y \; \{y | G(y) \leq 0\}$$

From H3) when this program has a solution, this solution is unique and at the optimum the constraint is tight.

Let us call $\eta(p)$ this solution. $\eta(p)$ is the supply vector of the production sector when production prices are p, and $\eta : \mathbb{R}^n_+ - \{0\} \to \mathbb{R}^n$ is the <u>supply function</u>. It is homogeneous of degree zero.

Let us remark that the above formulation rests upon a rough treatment of production since it only describes a regular aggregate constraint for consumption commodities and evades the study of the production of intermediate goods in several firms. However a more sophisticated description of the production possibilities would not basically modify the line of argument presented in the following.

II - THE STARTING POINT.

Let us consider an initial position of the economic system, —which takes place at time zero— in which :

- the production price system is $p(0)$
- the consumption price system is $q(0)$
- the tax vector is then $T(0) = q(0) - p(0)$
- the consumption vector of household h is $x_h(0)$
- the aggregate production plan is $y(0)$

Moreover, this initial position is supposed to be an equilibrium —relatively to the tax system $T(0)$— in the sense that

$$x_h(0) = x_h(q(0)) \qquad (\alpha)$$
$$y(0) = \eta(p(0)) \qquad (\beta)$$
$$\sum_{h=1}^{H} x_h(0) \leq y(0) \qquad (\gamma)$$

More precisely, one will suppose that constraints corresponding to (γ) are met :

$$\sum_{h=1}^{H} x_h(0) = y(0) \quad \ldots\ldots \quad (\delta)$$

When (α), (β) and (δ) will be met, one will say that the corresponding equilibrium is <u>tight</u>. If (γ) holds and (δ) does not hold, the equilibrium will be <u>non tight</u>.

Let us first remark, that the implementations of such an equilibrium requires that the government be able to disconnect consumption and production prices through consumption taxes and to operate a 100 % taxation of pure profits. These implicit assumptions on the set of policy tools have been lenghtily discussed otherwise, and one will limit ourself to this brief recall.

A second remark concerning this definition is that T(0) is considered a data. Another approach would have consisted in considering a set of given initial taxes $\overline{T}$ and wondering wether an equilibrium can be reached given these fixed taxes (existence problem). For such a problem a positive answer cannot be expected wathever $\overline{T}$. It follows that the tax vector $\overline{T}(0)$ associated with our initial equilibrium position cannot be any vector T, but we are not interested here in discovering the restrictions on T(0) which makes it compatible with an equilibrium.

One can now state additionnal assumptions. These assumptions are local assumptions, in the sense that they only concern characteristics of the system in a neighbourhood of the initial situation. In order to be distinguished from global assumptions, they will be denoted not by numbers but by greek letters.

Hα) x_h is continuously differentiable in a neighbourhood of q(0), $\forall$ h

Hβ) η is continuously differentiable in a neighbourhood of p(0).

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If all x_h are differentiable, X the aggregate demand function is also differentiable. We will denote $\overline{\overline{\partial X}}(0)$, the (n × n) matrix whose element in ℓ^{th} line, k^{th} column is $\left(\frac{\partial X_\ell}{\partial q_k}\right)_{(q(0))}$

Similarly $\overline{\overline{\partial \eta}}(0)$ will denote the (n × n) matrix whose element in ℓ^{th} line, k^{th} column is $\left(\frac{\partial \eta_\ell}{\partial p_k}\right)_{(p(0)}$. One knows that $\overline{\overline{\partial \eta}}(0)$ is a symetric matrix such that $p(0) \circ \overline{\overline{\partial \eta}}(0) = 0$. It follows that $\overline{\overline{\partial \eta}}(0)$ is at most of rank n-1.

Assumption Hγ) can then be stated, which asserts that $\overline{\overline{\partial \eta}}(0)$ is exactly of rank n-1.

Hγ) $\overline{\overline{\partial \eta}}(0)$ is of rank (n-1)

Hα), Hβ), Hγ) are not, strictly speaking, implied by H2), H3). Nevertheless they do not introduce severe restrictions in addition to H2),H3)(1).

III - A PRELIMINARY LEMMA.

Hβ) - Hγ) allows to prove the following useful lemma :

Lemma 1 :

Let us consider $V(0) = \{u \in \mathbb{R}^n \mid p(0) \circ u = 0\}$.

Then $\overline{\overline{\partial \eta}}(0)$ defines a one to one correspondence from V(0) onto V(0) denoted $\tilde{\partial}\eta(0)$.

The proof of the lemma proceeds as follows :

As $\overline{\overline{\partial \eta}}(0) = (\partial\eta(0)^T) \quad \overline{\overline{\partial \eta}}(0) \cdot p(0)^T = 0$. This means that $p(0)^T$ belongs to the kernel of the linear mapping defined by $\overline{\overline{\partial \eta}}(0)$. But $\overline{\overline{\partial \eta}}(0)$

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(1) The problem of differentiability of demand function has been discussed extensively elsewhere (see DEBREU, Econometrica 1972). This discussion could be transposed to supply functions. In case of demand functions, the only serious disturbing non-differentiabilities occur at prices in the neighbourhood of which the consumption of some commodity changes from zero to a positive quantity.

being of rank (n-1), the kernel is of dimension 1 and $\overline{\overline{\partial\eta}}(0)$, defines a one to one correspondence from a supplementary of the kernel – as is V(0)– onto its image (Im). It remains to prove that Im V(0) = V(0) ; which results from the fact that dim. Im V(0) = n-1 and that $\overline{\overline{\partial\eta}}(0)\circ u \in V(0)$ since $p(0)\circ\overline{\overline{\partial\eta}}(0) = 0$. Q.E.D.

Hence $\widetilde{\partial\eta}(0) : V(0) \to V(0)$, the restriction of $\overline{\overline{\partial\eta}}(0)$ to V(0) has an inverse which will be denoted $\widetilde{\partial\eta}^{-1}(0)$.

The intuitive content of the argument of the proof and of the consequence of lemma 1 must be emphasised :

$\overline{\overline{\partial\eta}}(0)\cdot p(0)^T = 0$ means that any small move of production prices in the direction of actual production price does not modify the supply vector. (A consequence of the homogeneity property). $\overline{\overline{\partial\eta}}(0)\circ u \in V(0)$ means that any small move of production prices leadsto moves in supply, the direction of which defines a vector normal to p(0) (an obvious geometric property).

The fact that $\widetilde{\partial\eta}(0)$ has an inverse means that any small move in supply, the direction of which is normal to p(0) can be obtained through a small modification of production prices, whose direction can be chosen normal to p(0). The fact that $\widetilde{\partial\eta}(0)$ is one to one means that the correspondence between directions of small moves in supply normal to p(0) and directions of production prices associated with such moves and normal to p(0) is one to one. As soon as one is aware of the normalization rule $\| p \| = C^{ste}$ implicit to the choice of production price changes normal to p(0), the two latter properties become intuitively appealing.

IV - SOME MORE PRELIMINARY DEFINITIONS.

In order to discuss the directions of tax reform one will introduce the following sets $K(0)$, $\overset{\circ}{K}(0)$, $Q(0)$, $\overset{\circ}{Q}(0)$

$$K(0) = \{a \in \mathbb{R}^n \mid a.x_h(0) \leq 0 \ , \ h = 1,\dots,H\}^{(1)} .$$

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(1) a is by definition a line vector. Hence a^T is a column vector.

Intuitively $K(0)$ is the set of prices systems, for which the cost of all consumption bundles $x_h(0)$ is smaller or equal to zero. Obviously any $a = \lambda\, q(0)$ belongs to $K(0)$ $\forall\, \lambda \geq 0$.

$\overset{\circ}{K}(0)$ will designate the interior of $K(0)$, i.e. the set of price systems for which the cost of all consumption bundles is strictly smaller than zero.

$$\overset{\circ}{K}(0) = \{a \in \mathbb{R}^n \mid a.x_h(0) < 0,\ h = 1,\dots,H\} .$$

From H1) it is clear that whatever the bundles $x_h(0)$, $\overset{\circ}{K}(0)$ is not empty (in order to lower the cost of all bundles (from $q(0)$) it suffices to raise the price of any "supplied commodity" or to lower the price of any "demanded commodity").

Finally, let us consider $Q(0)$, its interior $\overset{\circ}{Q}(0)$, its frontier $Fr\, Q(0)$

$$Q(0) = \{a \in \mathbb{R}^n \mid p(0)\circ\overline{\overline{\partial X}}(0)\circ a^T \leq 0\}$$
$$\overset{\circ}{Q}(0) = \{a \in \mathbb{R}^n \mid p(0)\circ\overline{\overline{\partial X}}(0)\circ a^T < 0\}$$
$$Fr Q(0) = \{a \in \mathbb{R}^n \mid p(0)\circ\overline{\overline{\partial X}}(0)\circ a^T = 0\}$$

$Q(0)$ can be given two related interpretations :

1. Let us consider a small change of consumption prices in the direction of a^T.
   The induced change in consumption is proportionnal to $\overline{\overline{\partial X}}(0)\circ a^T$. The value of this change expressed with production prices is $p(0)\circ\overline{\overline{\partial X}}(0)\circ a^T$. So $Q(0)$ is <u>the set of directions of consumption prices changes which imply changes in consumption whose value expressed with production prices is negative</u>.

2. Let us consider the budget surplus Δ as a function of p, q
   $\Delta(p,q) = (q - p)\circ X(q) + p\circ\eta(p)$. ($\Delta$ is the sum of receipts coming from consumption taxes and profit tax). One can check that $\Delta(p(0),q(0)) = 0$: in the initial equilibrium state, the government Budget is balanced. Let us consider however a small move of consumption prices, production prices being supposed to remain constant (generally, this does not define a feasible state). Taking into account $q.X(q) = 0$, it comes out $d\Delta = -\, p.dX$. Thus, $Q(0)$ also appears as the set of directions of tax changes, which all other things being equal, would preserve the Budget balance.

Let us notice, before pursuing further that the knowledge of K(0) and Q(0) requires information of different nature. K(0) is known as soon as the consumption bundles are known, when Q(0) depends on the set of all price-elasticities of the aggregate demand function, the evaluation of which requires sophisticated investigation.

In the following, the sets K and Q will sometimes be not indexed by time (here zero), but considered functions of production and consumption price vectors. In this way, without introducing additional notations, K(q) will designate $\{a \in \mathbb{R}^n \mid a \cdot x_h(q) \leq 0 \ , \ \forall\ h = 1,\ldots,H\}$ (so that K(0) is a notation for K(q(0))) and Q(p,q) will be: $\{a \in \mathbb{R}^n \mid p \cdot \overline{\overline{\partial X}}(q) \cdot a^T \leq 0\}$. (so that Q(0) is a notation for Q(p(0),q(0))).

All the elements of the model are now presented. We are in a position to give a more precise formulation of the problem studied here. Loosely speaking, our aim is to exhibit small tax changes which are first feasible, second, satisfactory in a Pareto sense or Pareto improving. For that, one will reason with infinitesimal moves (a natural idealisation of "small" moves) of the system. Relating these infinitesimal moves with infinitesimal moves of an exogenous variable called time —and denoted dt— allow defing directions of moves of variable z as $\frac{dz}{dt}$ and lead to formal definitions of "feasible" and "Pareto improving" directions of tax reform.

Precisely a direction of move of consumption prices denoted $\frac{dq}{dt}$, and a direction of move of production prices denoted $\frac{dp}{dt}$ will be said equilibrium preserving if :

$$\sum_{h=1}^{H} \frac{dx_h}{dt} \leq \frac{dy}{dt} \quad \text{with} \quad \frac{dx_h}{dt} = \overline{\overline{\partial x}}_h(0) \cdot \left(\frac{dq}{dt}\right)^T \ , \quad \frac{dy}{dt} = \overline{\overline{\partial \eta}}(0) \cdot \left(\frac{dp}{dt}\right)^T$$

The direction of move of prices $\frac{dq}{dt}$, $\frac{dp}{dt}$ will be said tight equilibrium preserving if

$$\sum_{h=1}^{H} \frac{dx_h}{dt} = \frac{dy}{dt} \quad \text{with} \quad \frac{dx_h}{dt} = \overline{\overline{\partial x_h}}(0) \circ \left(\frac{dq}{dt}\right)^T, \quad \frac{dy}{dt} = \overline{\overline{\partial \eta}}(0) \circ \left(\frac{dp}{dt}\right)^T$$

Thus, a tight equilibrium preserving direction of change tends to maintain the equality between demand and supply and not only to assure the inequality.

Similarly, directions of moves of consumption and production prices at time zero $\frac{dq}{dt}$ and $\frac{dp}{dt}$ will be said <u>strictly Pareto improving</u> if (1) :

1. $\frac{dq}{dt}$, $\frac{dp}{dt}$ are equilibrium preserving
2. $\frac{dV_h}{dt} = \left[\frac{\partial V_h}{\partial q}\right](0) \cdot \left(\frac{dq}{dt}\right)^T > 0$, $\forall$ h = 1,...,H

Hence, price changes in a strictly improving direction, tends to increase the welfare of all individuals.

B - STATEMENT OF RESULTS.

Feasible and Pareto improving moves of prices are characterized in section I et II. Results are commented in III.

I - FEASIBLE DIRECTIONS OF PRICE CHANGES.

One will always suppose that H1-H2) Hα)-Hβ) Hγ) are true

PROPOSITION I :

For any direction of consumption prices changes $\frac{dq}{dt}$ belonging to Q(0), one can find at least one direction of production prices changes $\frac{dp}{dt}$ such that $\left(\frac{dq}{dt}, \frac{dp}{dt}\right)$ be equilibrium preserving.

Moreover, If $\frac{dq}{dt} \in$ Fr Q(0) , the associated production prices direction of change is unique and $\left(\frac{dq}{dt}, \frac{dp}{dt}\right)$ is tight equilibrium-preserving.

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(1) Simply Pareto improving direction of moves would be defined by replacing 2 by 2':

2' : $\frac{dV_h}{dt} \geq 0$, $\forall$ h and $\exists$ h_o with $\frac{dV_{ho}}{dt} > 0$

Proof :

Let $\frac{dq}{dt} \in \mathrm{Fr}\, Q(0)$ be

$$\frac{dX}{dt} = \overline{\overline{\partial}} X(0) \circ \left(\frac{dq}{dt}\right)^T \text{ and } \quad p(0)\circ\frac{dX}{dt} = p \bullet \overline{\overline{\partial}} X(0) \circ \left(\frac{dq}{dt}\right)^T = 0.$$

Hence, $\frac{dX}{dt} \in V(0)$ and from Lemma 1, there exists $\frac{dp}{dt} = \tilde{\partial}\eta^{-1}(0)\circ\frac{dX}{dt}$.

It follows that $\frac{dy}{dt} = \overline{\overline{\partial}}\eta(0)\circ\left(\frac{dp}{dt}\right)^T = \overline{\overline{\partial}}\eta(0)\circ\tilde{\partial}\eta^{-1}(0)\,\frac{dX}{dt} = \frac{dX}{dt}$.

If $\frac{dq}{dt} \in \mathrm{Int}\, Q(0)$,

$\frac{dX}{dt} = \overline{\overline{\partial}} X(0)\circ\left(\frac{dq}{dt}\right)^T$ is such that $p(0)\circ\frac{dX}{dt} < 0$

One can take $U \in \frac{dX}{dt} + \mathbb{R}^n_+$ such that $p(0)\bullet U = 0$.

Using the same argument as above, it can be seen that $\frac{dq}{dt}, \frac{dp}{dt} = \tilde{\partial}\eta(0).U$ is equilibrium preserving but not tight equilibrium preserving.

Proposition I has a strong intuitive content : with respect to the discussion of p. 6, it means that if a small move of consumption prices is such that the value of the associated consumption changes, measured with the production prices, does not increase, —or equivalently is such that the State Budget be not affected in the way indicated p. 6, then the equilibrium of the system can be maintained through an adequate change of the production price system. It is worth of noting that the fact of belonging to Q(0) for a direction of change only removes one degree of freedom for the possible movements of the consumption prices vector As such a vector is a vector of $\mathbb{R}^{n-1}$ (taking into account the homogeneity of demand functions), it can be stressed, that proposition I implies, loosely speaking, that from any equilibrium the <u>system can move in (n-2) directions</u>.

Proposition II completes proposition I, by establishing the existence of small finite moves associated with tight equilibrium preserving directions of moves.

PROPOSITION II :

For any $a(0) \in Fr\ Q(0)$

There exists t_o and paths of prices $p(t)$, $q(t)$, $t \leq t_o$, such that :

. $\left(\frac{dq}{dt}\right)_{t=0} = a(0)$

. $p(t)$, $q(t)$, $x_h(q(t), \eta(p(t)))$ define tight equilibria $\forall\ t \leq t_o$

Proof :

Let us consider $F\ (Q)$ as a function of p and q (cf p. 7) and let $\nu(p,q)$ be the projection of $a(0)$ on the hyperplane $Fr\ Q(p,q)$.

Let the differential system be : $\frac{dq}{dt} = \nu(p,q)$ (1)

$$\frac{dp}{dt} = \tilde{\partial\eta}^{-1}(p) \cdot \overline{\overline{\partial X}}(q) \cdot \nu(p,q) \quad (2)$$

With Hα-Hβ) the second member of (1) and (2) is continuously differentiable. Hence a standart argument of existence (cf DIEUDONNE [1969]) allows to assert that the system has locally a solution.

Hence the conclusion. Q.E.D.

II - THE DIRECTION OF PARETO-IMPROVING PRICE CHANGES.

The directions of Pareto improving directions of move of prices can be characterized through propositions III and IV.

PROPOSITION III : (1)

For any direction of consumption prices changes $\frac{dq}{dt}$ belonging to $\overset{\circ}{\widehat{K(0)}} \cap Q(0)$, one can find at least one direction of production price changes $\frac{dp}{dt}$ such that $\left(\frac{dq}{dt}, \frac{dp}{dt}\right)$ be strictly Pareto improving.

Moreover, if $\frac{dq}{dt} \in \overset{\circ}{\widehat{K(0)}} \cap Fr\ Q(0)$, $\frac{dp}{dt}$ is unique and $\left(\frac{dq}{dt}, \frac{dp}{dt}\right)$ is tight equilibrium preserving.

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(1) As the reader will immediatly verify, a similar proposition applies to simply Pareto-improving changes, $\overset{\circ}{\widehat{K(0)}}$ being replaced by $K(0)$.

Proof :

Given proposition II, it is enough to prove that $\frac{dV_h}{dt} > 0$ $\forall$ h = 1,...,H, where V_h is the indirect utility function defined p. 2 and where $\frac{dV_h}{dt} = \left(\frac{\partial V_h}{\partial q}\right)_{q(0)} \circ \left(\frac{dq}{dt}\right)^T$ with $\left(\frac{\partial V_h}{\partial q}\right) = \left[\left(\frac{\partial V_h}{\partial q_k}\right)_{q(0)}\right]$

But our assumptions assure that $\left(\frac{\partial V_h}{\partial q}\right)_{q(0)} = \lambda_h\, x_h(0)$ (1) where λ_h is a strictly positive number which can be interpreted the individual value of income of h. Conclusions follows.

The content of proposition III is intuitively clear, if one reminds that any direction of price change belonging to $\overset{\circ}{\widehat{K(0)}}$ tends to decrease the cost of consumption bundles of all individuals.

Proposition IV gives a condition for the existence of strictly Pareto improving price changes, in terms of the position of the vector $p(0).\overline{\overline{\partial X}}(0)$ —vector of production costs associated with lowering all consumption prices of one "small" unit— and of the cone generated by consumption vectors.

PROPOSITION IV :

Let $\Lambda(0)$ be the cone generated by the consumption vectors $x_h(0)$

$$\Lambda(0) = \{x \mid x = \sum_{h=1}^{H} \lambda_h\, x_h(0)\ ,\ \text{for some}\ \lambda_h \geq 0\}.$$

If the vector (2) $-[p(0)\circ\overline{\overline{\partial X}}(0)]^T \notin \Lambda(0)$, there exist strictly Pareto improving directions of prices changes.

If in addition $[p(0)\circ\overline{\overline{\partial X}}(0)]^T \notin \Lambda(0)$, there exist strictly Pareto-improving directions of price changes which are tight equilibrium preserving.

Proof :

With respect to proposition III, it is enough to prove (for the first part) that $-[p(0)\circ\overline{\overline{\partial X}}(0)]^T \notin \Lambda(0) \Longrightarrow \overset{\circ}{\widehat{K(0)}} \cap Q(0) \neq \emptyset$

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(1) H2) and Hα) are needed.

(2) "If"could be replaced by "if and only if" as the reader will verify.

Let us suppose the contrary : $\overset{\circ}{\widehat{K(0)}} \cap Q(0) = \emptyset$. Then, from the separation theorem given in the appendix, there exists $\lambda_h \geq 0, \mu \geq 0$ such that $\sum_h \lambda_h x_h(0) + \mu\, p(0) \cdot \overline{\overline{\partial X}}(0) = 0$. One can rule out $\mu = 0$, which by the same theorem would imply $\overset{\circ}{\widehat{K(0)}} = \emptyset$ which is wrong (cf p. 6). Hence a contradiction is obtained. Using the same separation argument one sees that $[p(0) \cdot \overline{\overline{\partial X}}(0)]^T \in \Lambda(0) \iff \overset{\circ}{\widehat{K(0)}} \cap \mathrm{Fr}\, Q(0) = \emptyset$. Conclusion follows.
The following corollary(1) can be stated :

<u>COROLLARY 1</u> : If $[p(0) \cdot \overline{\overline{\partial X}}(0)]^T \in \Lambda(0)$, there exist Pareto improving directions of price changes, <u>all of them leading to non tight equilibria</u>.

Implications of the proposition IV for small but finite moves are given by proposition V.

<u>PROPOSITION V</u> :

If $-[(p(0) . \overline{\overline{\partial X}}(0)]^T \notin \Lambda(0) \cup -\Lambda(0)$, there exist a small but finite Pareto improving move, which is tight equilibrium preserving.

<u>Proof</u> :

One knows that the assumptions imply that $\overset{\circ}{\widehat{K(0)}} \cap \mathrm{Fr}\, Q(0) \neq \emptyset$. Let $a(0) \in K(0) \cap \mathrm{Fr}\, Q(0)$. One can apply proposition II above and consider along the path $p(t), q(t)$ (starting from $p(0), q(0)$ with $\left(\frac{dq}{dt}\right)_{(0)} = a(0)$), the quantities $(x_h(p(t)) \cdot \frac{dq}{dt}(t)\ h = 1, \dots, H)$. From the continuity of functions $x_h(p)$, $p(t)$, $\frac{dq}{dt}(t)$, one can conclude that for t small enough, all these quantities remain strictly negative. It follows that the utilities of all individuals strictly increase.

III - <u>COMMENTS AND COMPLEMENTS</u>.

a) The above analysis gives a criterion (Proposition IV) for determining wether a given equilibrium can be improved upon in the Pareto sense through "small" manipulations of the tax system. Testing this criterion in a given situation requires the knowledge of prices, (production prices) of quantities (consumption bundles of all individuals) —which are directly observable—.

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(1) Another corollary of proposition IV is that if T(0) is "small enough" there does not exist Pareto improving directions of tax change (since obviously $[p(0) . \overline{\overline{\partial X}}(0)]^T \in -\Lambda(0)$). This suggests that tight equilibria when such equilibria do exist with p(0) close to q(0) are second best optimal : an intuitive property which could be made rigorous.

and of elasticities of aggregate demand —which are not directly observable— (It is worth of noting that the knowledge of elasticities of individual demand are not needed. Elasticities of supply are not needed for testing the criterion but only for computing the effective tax change).

On the other hand, it is clear that those equilibria which cannot be improved upon by any tax manipulation, do not meet the above criterion : hence, characteristics of such states which are termed, conforming to the usual vocabulary, second best Pareto-optimal states, appear as a straightforward consequence and a joint product of this analysis.

COROLLARY 2 : In any second best equilibrium, $-[p(0)\cdot\overline{\overline{\partial X}}(0)]^T \in \Lambda(0)$
Equivalently, there exists $\lambda_h \geq 0$ s.t. $-[p(0)\cdot\overline{\overline{\partial X}}(0)]^T = \sum_{h=1}^{H} \lambda_h x_h(0)$.

The conditions are identical to those given by DIAMOND-MIRRLEES in their seminal article [1971] (formula (66)). Besides providing another proof, this makes it clear that such conditions do not rest on the use of a social welfare function, a concept foreign to the analysis attempted here.

b) However, as stressed in the introduction, the emphasis is put here on characterizing strictly Pareto improving directions of moves of a system in a given situation, rather than on characterizing situations which are Pareto optimal. If proposition IV gives a criterion for determining wether unanimously advantageous "directions of tax reform" exist, proposition III allows to exhibit such directions by selecting directions of consumption prices moves in $\overset{\frown}{K(0)} \cap Q(0)$ and adapting correspondingly the production price system. Three remarks will be made :

1. Giving an operational representation of the set K(0) raises computational problems which are slightly different according to wether the number of households is smaller or greater than the number of commodities . A view of these problems is provided in the appendix (1).

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(1) It is not surprising that loosely speaking the relative "importance" of the set of second best optima relatively to the set of tight equilibria depends crucially on the relative number of commodities and of households. The analysis given in the appendix provides a beginning of formal approach to this phenomenon.

2. Voluntarily, the attention has not been focused here on normalization problems. If productions prices are modified according to the implicit and specific normalization rule $\|p\| = \text{Cste}$, consumption prices movements are not governed by any a priori normalization constraint ; so that taxes — in the ordinary meaning of the term— are not unambiguously fixed. It is clear that the normalization rule for production prices could be modified and that a normalization rule for consumption prices could be imposed in order to meet any a priori requirement in this matter, without affecting the basic line of argument. For example, assuming the existence of an untaxed commodity — "labor"— , leads to economically meaningful interpretations. Additional insights on the tax system could possibly have been gained from such a normalization convention. They remain outside the scope of this study.

3. Raising a similar problem in a different — and simpler— context (one consumer, lump sum transfers feasible), DIXIT [1975] in a systematic investigation was able to obtain strong results : especially, he exhibited moves of the tax system (in terms of specific as well as ad valorem taxes), which where both (in some sense) "distorsions-reducing" and desirable. The reader will easily convince himself from the examination of $K(0) \cap Q(0)$ and proposition III that results of a similar type are quite unlikely to be obtained in this model, both because lump sum transfers are excluded and because of the distributional problems appearing in a many agents economy. This reinforces DIXIT's conclusion according to which the real problem is not "that there are few policies leading to partial welfare improvements...nor...that partial welfare improvements(1) are particularly difficult to characterize.." but "that some particular rules that were thought to be intuitively plausible by some economists turned out to be wrong". Let us add that familiarity with the analysis of direction of tax reform could be an appropriate way of developing correct intuitions in this field.

4. From Proposition III, it turns out that if $[p(0).\overline{\overline{\partial X}}(0)]^T \in \Lambda(0)$ the only way of obtaining a Pareto-improvement is to implement a non tight equilibrium, i.e. with DIAMOND-MIRRLEES vocabulary an inefficient equilibrium. This calls frr two remarks :

. Firstly, since no a priori restrictions are put on the tax system and on $\overline{\overline{\partial X}}(0)$ (but those resulting from homogeneity) equilibria when $p(0).\overline{\overline{\partial X}}(0) \in \Lambda(0)$ can actually occur.

. Secondly, such a property <u>does not contradict the efficiency property</u> which here straightforwardly holds in any second best equilibrium (cf H1). It only means that despite the need for efficiency in the final stage, <u>temporary inefficiencies may be necessary and unavoidable in the process of tax reform.</u>

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(1) Pareto improvements with our vocabulary.

c) The above analysis is a local analysis aimed at determining small moves of the tax pattern in the right direction and inducing small moves of the economy. However these small moves can be linked one with another in order to define changes of finite magnitude in the economic system.

Such connected moves obey differential equations,which can be straightforwardly exhibited from the local approach.

<u>COROLLARY 3</u> : Let $p(t)$, $q(t)$ be paths of production and consumption prices starting from $p(0)$, $q(0)$ and such that $\forall\ t \in [0,T]$

$$\frac{dq}{dt} \in \overset{\circ}{\widehat{K(p,q)}} \cap Fr(Q(p,q))$$

$$\frac{dp}{dt} = \widetilde{\partial\eta}^{-1}(p) \circ \overline{\overline{\partial X}}(q) \cdot \frac{dq}{dt} \quad (1)$$

Then, $x_h(q(t))$, $\eta(p(t),p(t),q(t)$ define a tight equilibrium $\forall\ t \in [0,T]$ and $V_h(q(t)$ is a strictly increasing function of t, $\forall\ h = 1,\ldots,H$.

However, we have just noticed that paths such those defined in Corollary 3 can be stopped before that Pareto improving changes fail to exist (when such changes unavoidably lead to non tight equilibria).

In the general case, temporary inefficiencies must be allowed, which makes the differential system slightly more complicated. For example such a system is given by corollary 4.

<u>COROLLARY 4</u> : Let $p(t)$, $q(t)$ be paths of production and consumption prices and $\lambda(t)$ be a positive number depending upon t, such that $\forall\ t \in [0,T]$

$$\frac{dq}{dt} \in \overset{\circ}{K}(p,q) \cap \tilde{Q}(p,q) \text{ with } \tilde{Q}(p,q) \begin{array}{ll} = Q(p,q) & \text{if } \eta(p) = X(q) \\ = \mathbb{R}^n & \text{if } \eta(p) > X(q) \end{array} .$$

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(1) $\widetilde{\partial\eta}$ is supposed to remain inversible along the path.

$$\frac{dp}{dt} = \tilde{\partial\eta}^{-1}(p)\circ[\overline{\overline{\partial X}}(q)\circ\frac{dq}{dt} - \frac{dy}{dt}p - \lambda \frac{dp}{dt}]$$

$$\frac{d\lambda}{dt} = \frac{p\circ\overline{\overline{\partial X}}(q)\circ\frac{dq}{dt}}{||p||^2}$$

Then $x_h(q(t))$, $\eta(q(t))$, $p(t)$, $q(t)$ define equilibria which are tight if and only if $\lambda(t) = 0$,

$V_h(q(t))$ is a strictly increasing function of t, $\forall\ h = 1,\ldots,H$

The reader will check that if $p(.)$ $q(.)$ satisfy the above equations

a) $p.\frac{dp}{dt} = 0$

b) Putting $\overline{\overline{\partial X}}(q)\circ\frac{dq}{dt} - \frac{d\lambda}{dt}p - \lambda\frac{dp}{dt} = \frac{dX'}{dt}$, then $p\circ\frac{dX'}{dt} = 0$

c) $\eta(p(t)) = X(q(t)) - \lambda(t)\,p(t) \qquad \lambda(t) \leq 0$

It seems that such differential systems are worth of being carefully studied, with reference to the different contexts in which they may be revelant tools of analysis.

- In the so called "economic theory of socialism", planning algorithms which have been proposed (see HEAL [1972]) rest upon the hypothesis that lump sum transfers are feasible. Such an assumption remains questionable in a socialist economy. If it were given up, finding optimal taxes would be a part of the optimal planning problem. Differential equations of corollaries II and III could be considered idealised formalisations of a tatonement planning procedure, where exchange of informations between the Center and the agents would be intended to discover elasticities (1).

- In a market economy, if demand and supply function are known not only locally but with some plausibility in a reasonably large interval (complete systems of demand functions such that those derived from the linear expenditure system of STONE (see for example SOLARI [1971] are supposed to provide such a knowledge for demand functions), the above differential system

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(1) Such a point was made clear to me by Y. YOUNES.

would have to be solved by the government, in order to implement a tax reform which would be not "small".

Considering the above differential system an idealization of a non tatonnement process in the spirit of the non-tatonnement adjustments of HAHN-NEGISHI, would also be possible, but not satisfactory with reference to Martin FELDSTEIN's analysis which makes it clear that a non tatonnement is not a correct formalisation of the process of tax reform in market economies.

CONCLUSION :

In conclusion, three possible extensions of the above analysis will be pointed out.

- Some parts of the analysis could be refined (introduction of a specific normalization rule for consumption prices, etc. consideration of "specific" or ad valorem texes, etc.).

- The analysis could easily be extended in order take into account one or several public goods. The differential systems of B-III.C would be modified accordingly. As argued above, they would remain a topic of independent interest.

- More generally, the principles of the method exposed here, distinguishing feasible from desirable moves, could be fruitfully applied to other second best situations.

APPENDIX

CONSTRUCTION OF THE SET $K(0) \cap Fr\ Q(0)$

Let us consider the case in which $K(0) \cap Fr(Q(0)) \neq \emptyset$ (equivalently $-[p(0)\circ\partial X(0)]^T \notin \Lambda(0)$ and $[p(0)\circ\partial X(0)]^T \notin \Lambda(0)$.
Let q be the number of extreme directions of the cone $\Lambda(0)$ generated by vectors $x_h(0)$. Obviously $q \leq H$.

In order to select directions in $K(0) \cap Fr\ Q(0)$, we will make additional requirements $\pi\circ a = 0$, a condition consistent with the normalization rule $\|\pi\| = C^{ste}$.

Let us notice that our assumptions assure that the vectors $x_{h_1}(0)\ldots x_{h_g}(0)$ (corresponding to extreme directions of Λ), $p(0)\circ\overline{\overline{\partial X}}(0) \overset{def}{=} W$ and $\pi(0)$ are linearly independant.

For constructing the set $K(0) \cap Fr\ Q(0)$, two cases have to be distinguished .

- In the case $q \geq n - 2$, extreme directions of this cone can be constructed as follows : taking any set $H\alpha)$ of $n-3$ indices chosen among q, one can consider C^q_{n-3} systems : $x_{h_i}\circ a = 0$ $(h_i \in H_\alpha)$, $W.a = 0$, $\pi\circ a = 0$

  Each such system has a one dimensional solution. Among these solutions some define half lines which are extreme directions of the polyhedron $K(0) \cap Fr\ Q(0)$.

  Any $a \in K(0) \cap Fr\ Q(0)$ is a convex combination of these extreme directions.

- If $q < n-2$, the system $\{x_h\circ a = 0$, $h = 1,\ldots,q$, $W.a = 0$, $\pi.a = 0\}$ defines a linear manifold of dimension $n-q-2$. The polyhedron has no extreme directions but only extreme faces. It cannot longer be described in a systematic way. However one can for example fix v coordinates of a

($v \geq n-q-2$) and proceed as above for extracting elements of the cone.

The incidence of the relative number of commodities and consumers on the construction of the set $K(0) \cap \mathrm{Fr}\, Q(0)$ is not a pure mathematical problem : it reflects more or less the economic idea that the degree of freedom in finding Pareto improving change of taxes increases when there are more tools.

SEPARATION THEOREM. (for cones of vertex 0).

Let $K_o, \ldots, K_{p-1}$ be p open convex cones and let K_p be a convex cone. $\bigcap_{o\ldots p} K_p = \emptyset$ if and only if there exists $q_o, \ldots, q_i, \ldots, q_p$ all of them non zero such that $\sum_{i=0}^{p} q_i = 0$, and $q_i \cdot x \leq 0$, $\forall\, x \in K_i$.

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APPENDIX II

Let us give an example of a situation in which any "small" Pareto improving price changes results in inefficient or non tight equilibrium :

There are three commodities 1,2,3, three households A,B,C.

At time zero the following tight equilibrium prevails

$$x_A(0) = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \qquad x_B(0) = \begin{pmatrix} 1,5 \\ 0,5 \\ -2 \end{pmatrix} \qquad x_C(0) = \begin{pmatrix} 0,5 \\ 1,5 \\ -2 \end{pmatrix} \qquad y(0) = \begin{pmatrix} 3 \\ 3 \\ -6 \end{pmatrix}$$

$$p(0) = (1,2,4) \quad , \quad q(0) = (1,1,1)$$

Furthermore the local characteristics of demand of households are such that all consumers have the same matrix of compensated demand $(\partial X)^{U=Cste} = A$ but have different income effect vectors $\frac{\partial X}{\partial R}$:

household A : $\quad (\overline{\overline{\partial X_A}})^{U=Cste} = A \qquad \frac{\partial X_A}{\partial R} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

household B : $\quad (\overline{\overline{\partial X_B}})^{U=Cste} = A \qquad \frac{\partial X_B}{\partial R} = \begin{pmatrix} 1 \\ -9/16 \\ +9/16 \end{pmatrix}$

household C : $\quad (\overline{\overline{\partial X_C}})^{U=Cste} = A \qquad \frac{\partial X_C}{\partial R} = \begin{pmatrix} 1 \\ 27/16 \\ -27/16 \end{pmatrix}$

where A is a negative semi definite matrix

$$A = \begin{pmatrix} -1 & 0,5 & 0,5 \\ 0,5 & -1 & 0,5 \\ 0,5 & 0,5 & -1 \end{pmatrix}$$

One can then check that

$$\overline{\overline{\partial X}}(0) = \begin{pmatrix} -6 & -1,5 & 7,5 \\ 1,5 & -5,25 & 3,75 \\ 1,5 & 3,75 & -5,25 \end{pmatrix}$$

$$q(0) \cdot \overline{\overline{\partial X}}(0) = (-3 \quad -3 \quad 6)$$

and

$$p(0) \cdot \overline{\overline{\partial X}}(0) = (+3 \quad +3 \quad -6)$$

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Then $p(0)\cdot\overline{\overline{\partial X}}(0) \in \Lambda(0)$. Hence there is no direction of price changes which is both feasible and Pareto improving.

Particularly, one can check that if either one decreases the prices of consumption goods or one increases the price of labor ; (which are obvious directions of Pareto improving movements) then the increase in labor demand from households is always greater than what is needed for producing the increase in consumer goods demand.

The reader might object that such examples rely on the existence of inferior goods.

Actually a slightly more complicated example can be given without inferior goods.

It is the following

- $x_A(0)$, $x_B(0)$, $x_C(0)$, $y(0)$, $p(0)$, $q(0)$ are as above.
- The local characteristics of demand of households are modified as above

$$A \;:\; (\overline{\overline{\partial X}}_A)^{U=Cste} = \mathcal{A} \qquad \frac{\partial X_A}{\partial R} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$B \;:\; (\overline{\overline{\partial X}}_B)^{U=Cste} = \mathcal{A} \qquad \frac{\partial X_B}{\partial R} = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$C \;:\; (\overline{\overline{\partial X}}_C)^{U=Cste} = \mathcal{B} \qquad \frac{\partial X_C}{\partial R} = \begin{pmatrix}0.8\\0.2\\0\end{pmatrix}$$

where $\mathcal{B}$ is the following semi definite negative matrix

$$\mathcal{B} = \begin{pmatrix} -0,5 & 0 & 0,5 \\ 0 & -2 & 2 \\ 0,5 & +2 & -2,5 \end{pmatrix}$$

Hence $$\overline{\overline{\partial X}}(0) = \begin{pmatrix} -3,9 & -1,2 & 5,3 \\ -0,6 & -4,8 & 5,12 \\ 1,5 & +3 & -4,42 \end{pmatrix}$$

And $$p(0)\cdot\overline{\overline{\partial X}}(0) = (0.9 \;,\; 1.2 \;,\; -2.1)$$

The latter vector is approximately 0.3 $(x_A(0) + x_B(0) + x_C(0))$ and is in the cone engendered by $x_A(0)$, $x_B(0)$, $x_C(0)$ (Actually it is a convex combination of $x_B(0)$, $x_C(0)$).

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