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ON TEMPORARY KEYNESIAN EQUILIBRIA

by

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ON TEMPORARY KEYNESIAN EQUILIBRIA*

by Jean-Michel GRANDMONT and Guy LAROQUE

One of the fundamental purposesof Keynesian theory is to present a model of the economy where transactions take place at prices that do not achieve the equilibrium of supply and demand as the classics understood it. This implies that, in such a model, short run adjustments must take place at least partly by quantity rationing instead of price movements.

Until recently, the research on Keynesian thinking has been done mainly within the framework of macroeconomic models pertaining to the neoclassical tradition. Money wages are assumed to display downward rigidities, and the labour markets are equilibrated by quantity rationing (unemployment). On the contrary, prices are supposed to move instantaneously on the markets for goods in order to match supply and demand. Accordingly, economic agents behave competitively on these markets. It has been shown, within the framework of this formalization, that, in some cases (destabilizing expectations, liquidity trap), there may exist no price system that would achieve an equilibrium of the economy in the classical sense (see, e.g., F. Modigliani (1963)[1]). Nethertheless, according to this line of thought, there is no fundamental difference at the conceptual level between the neoclassical and the keynesian models. It is this "neoclassical synthesis" that one finds in many macroeconomic textbooks.

After the works of Clower (1965), Leijonhufvud (1968), Patinkin (1949, 1965), the research on this topic has developed in a different direction. The classical axiom claiming that prices move instantaneously to match supply and demand is rejected. One is thus led to consider a polar case of the previous one and to study models which use the "fixed prices method" of Hicks (1965). Prices are assumed to be rigid in the short run. The allocation of resources is then achieved only by quantity rationing. Therefore, when making their choices, the agents will take into account the quantitative constraints that they perceive on the various markets (Barro and Grossman (1971), Grossman (1971, 1972), Solow and Stiglitz (1968)). This conceptual framework seems much richer than the previous one, for it allows to rationalize such concepts as the keynesian consumption function, the accelerator, or the existence of unvoluntary unemployment. It also permits to take into account such phenomena as the Phillips'curve (Iwai (1972, 1973)).

The foregoing studies were all made either in a macroeconomic framework, or in a partial equilibrium analysis. It seems therefore useful to reexamine the issue with the help of modern techniques of general equilibrium analysis. It is one of the purposes of the present work. This approach was made possible by the recent research on temporary competitive equilibrium models (Arrow and Hahn (1971), Grandmont (1970, 1971), Green (1971, 1972), Sondermann (1971), Stigum (1969, 1973)), and by some important contributions to equilibrium theory in case of price rigidities (Benassy (1973, 1974), Dreze (1973, b), Younes (1970, 1973)[2,3]). A previous attempt of this type was made by Benassy (1973), with different techniques.

The aim of this study is to present and compare the neoclassical and neokeynesian models within a unified framework. We shall argue that imperfect competition must be a central feature of the keynesian model. As a matter of fact, in the neoclassical tradition, prices are determined by the short run interaction of supply and demand, and the internal consistency of the model does not force to make more explicit how prices are set. On the contrary, once the fixed prices method is used, the mere logical consistency of the model requires that prices must be quoted by agents belonging to the system. We shall also emphasize an important feature that seems to have been underestimated on the previously quoted works, namely, the intertemporal character of production activities, and thus, the importance of producers'expectations regarding future effective demand in the determination of current wages and employment.

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In order to simplify the analysis, we shall consider a rudimentary economy composed of consumers-workers and of firms who exchange among themselves (consumption) goods, labor services and fiat money. In period t, firms combine goods available at the outset of the period and labor services, to produce goods that will be available at the begining of period t+1. We will exclude from the analysis long term planning considerations, and thus will not explicitly introduce capital goods. Moreover, there will be no financial system that would enable firms to find external funds. Finally, we shall ignore the possible existence of a stock market or of dividend distribution. The latter restrictions seem unimportant.

We shall study first the neoclassical interpretation of the keynesian model for such an economy. In this case, all agents behave as price takers. On the other hand, money prices and wages are free to move at date t to match supply and demand, but money wages cannot fall below some a priori given values. When a wage hits its minimum value, the corresponding labor supply is rationed. We shall prove the existence of such an equilibrium under the assumptions that are commonly used in the study of temporary competitive equilibrium models (continuous price expectations which do not depend "too much" on the current price system). We shall also show that, under the same assumptions, a competitive equilibrium (i.e., without rationing) exists in this economy when there is no downward wage rigidity. Under some conditions which are weak from the neoclassical point of view, stating essentially that the marginal real productivity of labor services is positive on the domain of feasible allocations, it can be shown that wages must be positive at a competitive equilibrium. It follows that, if all these conditions are satisfied, unemployment would not exist if minimum money wages were low enough.

We shall study in the second part of the paper a keynesian model with imperfect competition. To fix the ideas, we shall postulate that prices are fixed by sellers. Accordingly, in the (very) short run, firms choose the p ices of their outputs, while workers choose the wages at which they would like to work [4]. These prices are quoted at the outset of period t. Then the adjustment of the markets for goods and labor services is achieved

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by quantity rationing (money is not rationed). It is important to notice that the agents base their decision at date t partly on their expectations about the future state of the economy. In particular, firms must forecast given sufficient conditions for the existence of a short run equilibrium with rationing, we shall analyse the possible sources of unemployment in this model. We shall find that there are some cases (when the producers' expectations concerning future effective demand are pessimistic) where there may be unvoluntary unemployment at all positive money wages[5].

The remainder of the paper is devoted to the formal treatment of the models, and to a discussion of their respective properties. In section 1, we describe the assumptions and concepts that are common to both models. We then examine in section 2 the neoclassical version of the keynesian model, and in section 3, a keynesian model with imperfect competition. A discussion of the models together with suggestions for future research are presented in section 4, while all proofs are gathered in sections 5 and 6.

1. DEFINITIONS AND ASSUMPTIONS.

We gather, in this section, all definitions and assumptions that are used in both models.

We consider an economy at date t. The agents who meet at that date are producers, indicated by j in the finite set J, and consumers-workers indicated by i in the finite set I. They exchange among themselves (consumption) goods indicated by k in the finite set K, labor services indicated by h in the finite set H and fiat money. We shall denote by $q \in R^K$ a vector of goods, $\ell \in R^H$ a vector of quantities of labor services, and $m \in R$ a quantity of money. By definition $N = K \cup H$. To simplify, N will also represent the number of elements in the set N. Accordingly, the commodity space is R^{N+1}. The associated monetary prices will be $p \in R^K$, $w \in R^H$ and 1. A price system is described by $s = (p, w, 1) \in R^{N+1}$.

We define [6] : $\bar{S} = \{s = (p,w,1) \in R^{N+1} \mid s \geq 0\}$

$$S = \{s \in \bar{S} \mid p >> 0\}$$

We shall choose a very simple representation of production activities. In order to focus the attention on short run problems, we will neglect the interdependance of short run and long run decisions and will limit the firms'planning horizon to one period. In addition, there will be no financial system, nor dividends distribution. Accordingly, the j-th producer's activity in period t is to combine inputs of goods and labor services to get outputs of goods available at date t+1. Such a representation encompasses storage activities. The production possibilities perceived by the j-th producer at date t are represented by a subset T_j of $R_+^K \times R_+^H \times R_+^K$. A productionpplan is described by $y = (q_1 , \ell, q_2)$ T_j , where q_1 and ℓ are inputs of goods and labor services, and q_2 is the expected output available at date t+1. We postulate [7] for every j :

(a.1) T_j <u>is convex, closed and</u> $0 \in T_j$.

(a.2) <u>For every bounded subset</u> B <u>of</u> $R_+^K \times R_+^H$, <u>the set</u> $\{q_2 \in R_+^K \mid (q_1, \ell, q_2) \in T_j , (q_1, \ell) \in B\}$ <u>is bounded</u>.

At date t, the j-th firms owns a stock of goods q_j $(t-1) \in R_+^K$ that was produced during the previous period, and an amount of money $m_j(t-1) \in R_+$. Its endowment of commodities is thus $e_j(t) = (q_j(t-1), 0, m_j(t-1)) \in R_+^{N+1}$.

Let us look at the i-th consumer $(i \in I)$, at date t. He consumes $q \in R_+^K$, sells the labor services $\ell \in R_-^H$ and keeps an amount of money $m \in R_+$. We shall assume, to simplify, that he cannot store goods. We will describe by $\ell_i^* \in R_-^H$ the maximum supply of labor services [8] of consumer i. By definition, $L_i = \{\ell \in R^H \mid \ell_i^* \leq \ell \leq 0\}$ and $H_i = \{h \in H \mid \ell_i^* \neq 0\}$. At each date, the vector $x = (q,\ell)$ must belong to $X_i = R_+^K \times L_i$. This implies that a typical consumer can survive without working, which is obviously a strong assumption.

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The consumer's planning horizon is limited to period t+1. The i-th consumer's preferences are thus defined on $X_i \times X_i$. By assumption, for every i,

(b) The preferences of consumer i can be represented by a function $u_i : X_i \times X_i \to R$ that is continuous and semi-strictly quasi-concave [9]. Further, $u_i(q_1, \ell_1, q_2, \ell_2)$ is increasing with respect to q_1 and q_2, non decreasing with respect to ℓ_1 and ℓ_2.

Since our consumer cannot store goods, his resources at date t are only composed of his cash balance $m_i(t-1) \in R_+$, and are thus described by $e_i(t) = (0, 0, m_i(t-1)) \in R_+^{N+1}$.

2. THE NEOCLASSICAL INTERPRETATION.

We assume in this section that all economic agents act as price-takers. Moreover, prices of goods are supposed to react rapidly enough in the short run to match supply and demand, through, for instance, some tatônnement process. On the other hand, money wages display downward rigidities : they cannot fall below some values described by $w^* \in R_+^H$. The constraints $w \geq w^*$ may be institutionaly given (minimum wages law) or may be set by the workers themselves [10]. By definition,

$$S^* = \{s = (p,w,1) \in S \mid w \geq w^*\}$$

$$\bar{S}^* = \{s = (p,w,1) \in \bar{S} \mid w \geq w^*\}$$

Then, if $w_h^* > 0$, and if, during the adjustment process, w_h hits the minimum value w_h^*, the corresponding labor market is equilibrated by rationing of the labor supply (underemployment). This rationing is brought about by means of a signal perceived by some (or all) workers that expresses, for each of them, the maximum amount of labor that he is allowed to supply on this particular market. On the other hand, when $w_h^* = 0$, the equilibrium of the corresponding labor market must be reached only by the price mechanism.

Therefore, in this model, the consumers'choices are function of the current price system, of the quantitative signals that they perceive if they are rationed on some labor markets, and of their knowledge of the past. On the other hand, producers are never rationed in this model, and thus base their decisions only upon their knowledge of the current price system and of the past states of the economy.

2.1. The model.

We begin with the consumers'behaviour. At date t, the i-th consumer must choose an action $a = (x,m)$ in $A_i = X_i \times R_+$ that specifies his consumption, his labor supply and the amount of money that he wishes to keep until the next period. In order to make his choice, the consumer must forecast the state of the economy at date t+1. In this model, it is enough for him to forecast the price system $s_2 \in S$ that will prevail, and the maximum amount of labor $\xi_2 \in L_i$ that he will be allowed to supply. For simplicity, this forecast is certain and is described by a point of $S \times L_i$. It depends upon the consumer's knowledge of the past and on the signals currently perceived. Since the past is fixed in the analysis, its influence on expectations is not explicited. On the other hand, by assumption, the signals currently perceived by the consumer are the current price system and quantity signals on some labor markets in case of rationing. We shall assume however that the consumer's forecast only depends upon the current price system. We shall come back to this assumption in the discussion of our concept of equilibrium [11]. Accordingly, the i-th consumer's expectations are described by a mapping ψ_i taking S^* into $S \times L_i$.

It is convenient to represent the consumer's behaviour as a two-steps procedure. Let $s_1 \in S^*$ be a price system quoted at date t. Let N_i , a (may be empty) subset of H, stand for the set of labor markets on which our consumer perceives a quantitative signal, and let $\xi_1 \in R_-^{N_i}$ be the corresponding signals. Then, the consumer's forecast is $\psi_i(s_1) = (s_2, \xi_2)$, where $s_2 = (p_2, w_2, 1)$. Now, for any action $a = (x_1,m) \in A_i$, let $v_i(a,s_1)$ be the maximum of $u_i(x_1, x_2)$ when $x_2 = (q_2, \ell_2)$ varies in X_i subject to $p_2 \cdot q_2 + w_2 \cdot \ell_2 = m$ and $\ell_2 \geq \xi_2$. Then, the consumer will choose an action

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$a = (x_1, m) \in A_i$, with $x_1 = (q_1, \ell_1)$, so as to maximize $v_i(a, s_1)$ subject to $s_1.a = s_1.e_i(t)$ and $\ell_{1h} \geq \xi_{1h}$ for every $h \in N_i$. The set of optimal actions is denoted $\alpha_i(s_1, \xi_1 , N_i)$.

We proceed now to the study of the producer j at date t. He must choose an action $a = (q_1, \ell_1, m)$ in $A_j = \text{Proj } T_j \times R_+$, where Proj T_j is the projection of T_j on $R_+^K \times R_+^{H^J}$. Here again, the producer must forecast the state of the economy at date t+1, that is, the vector of prices of goods that will then prevail. We shall again assume that this forecast is certain in order to avoid the problems involved in the definition of a satisfactory criterion for a firm operating under uncertainty. If we do not formally take into account the influence of the past upon expectations, the producer's forecast is function of the current price system alone, and can therefore be described by a mapping ψ_j taking S^* into R_+^K .

Given the current price system $s = (p, w, 1) \in S^*$, the producer will try to maximize the money value of the firm at date t+1. Equivalently, he will maximize the expected profit $\psi_j(s).q_2 - p.q_1 - w.\ell_1$ subject to $a = (q_1, \ell_1 , m) \in A_j$, $y = (q_1, \ell_1, q_2) \in T_j$ and $s.a = s.e_j(t)$, y and a being the unknowns of the problem. This yields a set of optimal actions $\alpha_j(s)$.

We next give a formal definition of equilibrium. In order to do that, we must specify when the workers perceive quantitative signals on the labor markets. First, it is natural to impose that no rationing occurs on the market h when $w_h > w_h^*$ or $w_h^* = 0$. Second, no constraint will be perceived by the i-th consumer on the labor market h if he does not participate in that market, i.e., if $\ell_{ih}^* = 0$. In other words, we shall impose $N_i \subset H_i$. Finally, we shall assume that a consumer does not perceive a quantitative signal on a given market if he is not rationed on that market. Formally, the price system $s = (p,w,1) \in S^*$, the actions $a_j \in A_j$ and $a_i \in A_i$, the (may be empty) sets $N_i \subset H_i$ and the signals $\xi_i \in R_-^{N_i}$ $(i \in I, j \in J)$ define a <u>neoclassical equilibrium with rationing</u> at date t if :

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(E.1) $\sum_i (a_i - e_i(t)) + \sum_j (a_j - e_j(t)) = 0.$

(E.2) $a_j \in \alpha_j(s)$ <u>for every</u> j, <u>and</u> $a_i \in \alpha_i(s,\xi_i,N_i)$ <u>for every</u> i.

(E.3) $w_h > w^*_h$ <u>or</u> $w^*_h = 0$ <u>implies</u> $h \notin N_i$ <u>for every</u> i.

(E.4) ξ_i <u>is the projection of</u> a_i <u>on</u> R^{N_i} <u>for every</u> i.

It must be noted that this definition does not specify the distribution of unemployment among workers : the rationing scheme is arbritrary. Given the present specification of the model, no particular scheme seems more appropriate. One can however study the existence of an equilibrium corresponding to a priori given rationing schemes.

We shall focus the attention on three cases.Given h, let $I_h = \{i \in I \,|\, \ell^*_{ih} \neq 0\}$. First, we can impose on the rationing scheme to be <u>uniform</u> [12] :

(E.5) $h \in N_i \cap N_g$ <u>implies</u> $\xi_{ih} = \xi_{gh}$ <u>for every</u> i <u>and</u> g <u>in</u> I_h .

Rationing on the labor market h may also be implemented according to <u>some ordering</u>. Let $\succ_h$ be an order relation defined on I_h . Then $i \succ_h g$ means that the consumer i must be rationed before the consumer g. We can impose that a worker g is constrained only if all workers i such that $i \succ_h g$ are fully rationed, that is :

(E.6) $h \in N_g$ <u>implies</u> $h \in N_i$ <u>and</u> $\xi_{ih} = 0$ <u>for all</u> $i \in I_h$
<u>such that</u> $i \succ_h g$.

Finally, we may impose on the rationing on the market h to be <u>proportional</u> to the workers'labor supply if they were not constrained on that market, i.e., to their effective labor supply. In order to make this concept of proportional rationing meaningful, we assume that the consumers' utility functions are strictly quasi-concave [13]. Formally, consider an equilibrium satisfying (E.1)-(E.4). For every i such that $h \in N_i$, let $N'_i = N_i \setminus \{h\}$, and consider $\xi'_i \in R^{N'_i}$ that is obtained from ξ_i by dropping the component ξ_{ih}. Under the assumption of strict quasi-concavity of

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preferences, the set $\alpha_i(s,\xi'_i, H'_i)$ reduces to a single point $\tilde{a}_i = (\tilde{q}_i, \tilde{\ell}_i, \tilde{m}_i)$. By definition, the i-th consumer's effective labor supply on the market h is equal to $\tilde{\ell}_{ih}$. Then, proportional rationing on the market h means [14] :

(E.7) <u>If</u> $h \in N_g$ <u>for some</u> $g \in I_h$, <u>one has</u> $h \in N_i$ <u>for all</u> $i \in I_h$, <u>and there exists a real number</u> $0 \leqslant \beta_h \leqslant 1$ <u>such that</u> $\xi_{ih} = \beta_h \tilde{\ell}_{ih}$ <u>for all</u> $i \in I_h$.

<u>Remark</u>. We assumed that the consumers'expectations were independent of the quantitative constraints that they may perceive on the current labor markets. We wish to discuss this assumption in connection with an important feature of our concept of equilibrium, that is, the assumption that workers do not receive a quantitative signal if they are not rationed.

Assume that expectations depend upon perceived constraints on the labor markets. Then, given $s_1 \in S^*$, $N_i \subset H$ and $\xi_i \in R_-^{N_i}$, the i-th consumer's forecast $(s_2, \xi_2) \in S \times L_i$ is denoted $\psi_i(s_1,\xi_i,N_i)$. For each $a = (x_1,m) \in A_i$, let $v_i(a,s_1,\xi_i,N_i)$ be the maximum of $u_i(x_1,x_2)$ when $x_2 = (q_2, \ell_2)$ varies in A_i subject to $p_2.q_2 + w_2.\ell_2 = m$ and $\ell_2 \geqslant \xi_2$. The set $\alpha_i(s_1, \xi_i, N_i)$ is defined as before as the set of actions $a = (q_1, \ell_1, m) \in A_i$ that maximize $v_i(s_1, \xi_i, N_i)$ subject to $s_1.a = s_1.e_i(t)$ and $\ell_{1h} \geqslant \xi_{ih}$ for all $h \in N_i$.

Suppose that we keep the assumption that workers do not receive a quantitative signal on a labor market if they are not rationed. Then equilibrium at date t is defined by conditions (E.1)-(E.4) above. But, when trying to prove the existence of such an equilibrium, one is confronted to a serious problem. For the consumers'behaviour can display discontinuities when one goes from a situation where is no rationing to a situation where a quantitative constraint is perceived. Of course, the problem disappears if expectations are assumed independent of rationing.

Another solution can be found by changing the concept of equilibrium. One can assume that workers do perceive signals on the labor markets

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even if they are not rationed. This type of solution was implemented by J.P. Benassy in his thesis (1973, 1974). Formally, this amounts to saying in the above definition of an equilibrium that $N_i = H_i = \{h \in H \mid \ell^*_{ih} \neq 0\}$ for all i and to replace (E.3) and (E.4) by :

(E.3 bis) $w_h > w^*_h$ or $w^*_h = 0$ implies $\ell_{ih} > \xi_{ih}$ for all $i \in I_h$.

The results below are then valid with trivial changes, provided that, for every i, $\psi_i(s, \xi_i, H_i)$ is continuous with respect to $s \in S^*$ and $\xi_i \in R_-^{H_i}$. The techniques of the proofs are unchanged.

2.2. An Existence Theorem.

Here are sufficient conditions to insure the logical consistency of the model.

THEOREM 1. Assume $\sum_j q_j(t-1) \gg 0$, $\sum_i \ell^*_i \ll 0$, $\sum_i m_i(t-1) > 0$, and for every i and j,

(1) ψ_j is a continuous function.

(2) For every sequence $s^r \in S^*$ such that $\lim \|s^r\| = +\infty$, $\lim s^r / \|s^r\| = (\bar{p}, \bar{w}, 0)$ with $\bar{w} = 0$, one has $\lim \psi_j(s^r) / \|s^r\| = 0$.

(3) ψ_i is a continuous function.

(4) The image of S^* by ψ_i is contained in a compact subset of $S \times L_i$.

Then, there exists a neoclassical equilibrium with rationing.

COROLLARY. Under the assumptions of the theorem, there exists an equilibrium satisfying (E.5) or (E.6). If, in addition, the consumers'utility functions are strictly quasi-concave, there exists an equilibrium satisfying (E.7).

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Assumption (4) is commonly used in temporary competitive equilibrium models. Together with $\sum_i m_i(t-1) > 0$, it makes sure that a "real balance effect" appears when some prices of goods tend to zero. In the presence of $\sum_i \ell_i^* << 0$, it guarantees that an excess demand appears for r large enough for every sequence s^r that tends to infinity such that $\lim s^r / \|s^r\| = (\bar{p}, \bar{w}, 0)$, with $\bar{w} \neq 0$. The purpose of assumption (2) is to obtain the same result when $\bar{w} = 0$.[15] This set of conditions implies the existence of a finite equilibrium price system, that is, prevents the price of money from becoming zero.

From (E.3) of the definition of an equilibrium, the foregoing theorem asserts the existence of a competitive equilibrium when $w^* = 0$.[16] It is interesting in that case to have conditions implying that money wages are positive at this equilibrium. To simplify, assume that for every j, the set T_j is defined by a production function F_j taking $R_+^K \times R_+^H$ into R_+^K :

$$T_j = \{(q_1, \ell_1, q_2) \mid (q_1, \ell_1) \in R_+^K \times R_+^H,\ q_2 \in R_+^K,\ q_2 \leqslant F_j(q_1, \ell_1)\}.$$

Assume further that for all actions $a_j \in A_j$ and $a_i \in A_i$ such that $\sum_j (a_j - e_j(t)) + \sum_i (a_i - e_i(t)) = 0$, and for every $h \in H$, there exists a producer j such that one of the left hand partial derivatives $\partial F_{jk} / \partial \ell_{1h}$ $(k \in K)$ is positive. In other words, the marginal physical productivity of labor is always positive on the set of feasible states of the economy. It is then clear that, if $\psi_j(s) >> 0$ for all $s \in S^*$ and $j \in J$, any competitive equilibrium is such that $w >> 0$.

To sum up, we have shown that an equilibrium with rationing exists provided that expectations do not depend too much on the current price system. The purpose of this assumption is to ensure the existence of a real balance effect in the economy. On the other hand, when expectations are strongly influenced by the current price system, one can find nonpathological examples where an equilibrium does not exist.[17] Thus, this result confirms a conjecture that is often made in the discussion of keynesian models : there are cases of "destabilizing expectations" where the logical consistency of the classical model (case $w^* = 0$) is not guaranteed. But one sometimes finds in the literature that downward wage rigidities will then restablish the consistency of the system. Theorem 1 shows that this conjecture is false, for we need exactly the same assumptions with or without downward wages rigidities. The need for assumptions of this type on expectations comes from the very nature of the Walrasian model : the price system is allowed to vary widely (in particular, it may go to infinity) during the tatonnement process. We shall see later on that such an assumption on expectations is no longer needed when one uses the "fixed price method".

The second important finding is that under the assumptions of the theorem, when $w^* = 0$, a competitive equilibrium (i.e. without rationing) always exists,with positive money wages, if the marginal real productivity of labor is positive on the set of feasible states of the economy. The latter condition is not really a restriction, from a neoclassical point of view, in developed economies with enough capital. Thus, we find, as the classics did, that unemployment can be removed in this model by a sufficient decrease of money wages. However, even if one accepts the logic of the model, one cannot claim that wages <u>should</u> be decreased for efficiency reasons, for a temporary competitive equilibrium does not in general display any reasonable optimality properties. This is due to the fact that all agents make decisions at date t in function of their expectations about the future which may be completely false. In particular, it is easily checked that the level of money wages at a competitive equilibrium may be quite low when the producers'<u>expectations</u> about the prices of their products are low themselves. This wage level cannot be considered as better than any other.

Finally, we would like to emphasize, after many others, a serious shortcoming of the above model, that is related to the interpretation of the constraints $w \geqslant w^*$. If one assumes that these constraints describe downward wage rigidities, in which case w^* is equal to the wages that prevailed in the previous period, the model yields the embarassing conclusion that persistent unemployment cannot be observed with rising wages, which is contrary to the facts. In order to solve this problem, one can consider that w^* is set by the workers in each period and revised

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in function of the evolution of the economy. But then, one is led to lend to the workers a price making behaviour. In the same spirit, one should admit a similar behaviour from the part of producers. It is precisely a model of this kind that we are going to study in the second part of this paper.

3. A KEYNESIAN MODEL WITH IMPERFECT COMPETITION.

We now consider a different functioning scheme of the economy. We assume that the agents are no longer price takers but behave as price makers. To fix the ideas, we postulate that the agents set the prices of the commodities that they sell. Therefore producers choose the prices of their outputs, while workers choose the wages at which they would like to work. By assumption, the price system which results of these choices is fixed at the outset of the market of date t. The equilibrium at date t is then achieved only by quantity rationing on the markets for commodities, with the exception of the money market. Thus, the following model must be interpreted as a very short period model.

Before describing the model, we must make more precise a few concepts. In the sequel, a commodity will be defined by its physical characteristics and by the agent who is able to sell it on the market. The set of goods K is thus partitioned into nonempty disjoint sets K_j $(j \in J)$, where K_j represents the set of products that the producer j can sell on the market. Two goods belonging to K_j and $K_{j'}$ can of course display the same physical characteristics. In the same spirit, the set H of labor services is the union of the nonempty sets $H_i = \{h \in H \mid \ell^*_{ih} \neq 0\}$ $(i \in I)$ that were defined in the first part of the paper. Further, the sets H_i are now assumed to be pairwise disjoint. This set of assumptions leads us to postulate the following conditions. First, when we write a production plan of the firm j, $(q_1, \ell_1, q_2) \in T_j$, it must be understood that q_2 is a vector of $R_+^{K_j}$: T_j is a subset of $R_+^K \times R_+^H \times R_+^{K_j}$. On the other hand, when considering $e_j(t) = (q_j(t-1), 0, m_j(t-1))$, we always assume that $q_{jk}(t-1) = 0$ for all k in $K \setminus K_j$.

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3.1. <u>The Model.</u>

We consider the economy at date t and suppose that the agents already have quoted the prices that they control. Accordingly, the producer j quoted $p_j^* \in R_+^{K_j}$, while the consumer i announced $w_i^* \in R_+^{H_i}$. We shall denote the resulting price system by $s^* = (p^*, w^*, 1) \in R_+^{N+1}$, where $p^* = (p_j^*)$ and $w^* = (w_i^*)$.

We first look at the producer j. He must choose an action $a = (q_1, \ell_1, m) \in A_j$, where A_j is the intersection of Proj $T_j \times R_+$ that was previously defined, and of $\{(q_1, \ell_1, m) \mid q_{1k} \leq q_{jk}(t-1)$ for every $k \in K_j\}$. The new constraint which appears in the definition of the set of feasible actions reflects the fact that producer j cannot be a net buyer of his own products, since he is the only producer of these goods.

The choice of an action by firm j will depend upon the signals received from the market (the fixed price system, and quantitative signals in case of rationing) and on the producer's expectations about the future effective demand for his products. We shall assume have again that the producer's expectations do not depend on the quantitative signals received in case of rationing. We shall describe the producer's forecast of the effective demand for his products at date t+1 by a function $\rho_j : R_+^{K_j} \rightarrow R_+$. Then, given $q \in R_+^{K_j}$, $\rho_j(q)$ represents the maximum proceeds that the producer expects at date t to get from the sale at date t+1 of the quantity $q \in R_+^{K_j}$. One can imagine that $\rho_j(q)$ is the result of the following process. Given his expectations about the behaviour of the other agents at date t+1, the producer tries to forecast the set of prices $p \in R_+^{K_j}$ that will allow him to sell exactly the quantity $q \in R_+^{K_j}$ at that date, taking into account the possible rationing of supply or demand. One can reasonably assume that this set is closed. On the other hand, if $q_k > 0$ for some $k \in K_j$, the associated component p_k must be bounded, for the product $p_k\, q_k$ cannot exceed the total wealth of the economy at date t+1. Finally, if $q_k = 0$, the corresponding component p_k can be chosen arbitrarily between 0 (included) and $+\infty$: the demand for this good will then be rationed.

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Under these conditions, given $q \in R_+^{K_j}$, there always exists a set $\pi_j(q) \subset R_+^{Kj}$ of prices that maximize the sale's proceeds p.q. Then, by definition, $\rho_j(q) = p.q$ for all $p \in \pi_j(q)$.[18]

We can now describe precisely the j-th producer's behaviour. Let N_j , a may be empty subset of N, be the set of markets on which the producer receives quantitative signals, described by $\xi \in R^{Nj}$. By definition, ξ represents constraints on the net exchanges of the producer. Further, $\xi_n \leq 0$ when $n \in N_j \cap K_j$ for, then, ξ_n is a constraint on the producer's excess supply of the good. Finally, $\xi_n \geq 0$ when $n \in N_j \setminus K_j$ for, then, ξ_n represents a constraint on the producer's net demand. Given N_j and such a $\xi \in R^{Nj}$, the producer will choose an action $a = (q_1, \ell_1, m) \in A_j$, a production plan $y = (q_1, \ell_1, q_2) \in T_j$ and an (expected) vector of sales at date t+1, $q \in R_+^{Kj}$, so as to maximize the expected profit $\rho_j(q) - p^*.q_1 - w^*.\ell_1$ subject to :

(i) $0 \leq q \leq q_2$

(ii) $s^*.a = s^*.e_j(t)$

(iii) $\xi_k \leq q_{1k} - q_{jk}(t-1)$ for every $k \in N_j \cap K_j$

(iv) $q_{1k} \leq \xi_k$ and $\ell_{1h} \leq \xi_h$ for all $k \in N_j \cap (K \setminus K_j)$ and all $h \in N_j \cap H$.

The constraint (i) expresses the fact that sales at date t+1 cannot exceed the available output,[19] while (ii) is the budget constraint of period t. Finally, (iii) and (iv) describe how the producer's net exchanges are constrained on the markets for commodities in N_j . The set of optimal actions corresponding to this problem is denoted $\alpha_j(\xi_j, N_j)$.

Let us study now the consumer i. He must choose an action in $A_i = X_i \times R_+$ in function of the quantitative signals that he receives from the market in case of rationing. As before, we assume that the consumers'expectations do not depend on the constraints that they perceive in case of rationing. In order to make a decision, the consumer i must forecast the prices of goods that will be quoted by the producers at

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date t+1, say $p_{i2} \in R_+^K$, as well as the maximum amount of these goods that he will be allowed to buy, say $\xi_{i2} \in R_+^K$. In addition, the consumer i must forecast the maximum income that he can receive if he decides to sell at date t+1 the labor services $\ell_2 \in L_i$, say, $\rho_i(\ell_2) \geq 0$. This defines an expected income function $\rho_i : L_i \to R_+$, that can be justified by the same arguments as in the case of producers.[20]

Here again, it is convenient to look at the consumer's behaviour as a two stages procedure. Given $a = (x_1, m_1)$ in A_i, let $v_i(a)$ be the maximum of $u_i(x_1, x_2)$ when $\mathbf{x}_2 = (q_2, \ell_2)$ varies in X_i subject to $p_{i2}.q_2 + m_2 = m_1 + \rho_i(\ell_2)$ and $q_2 \leq \xi_{i2}$, where m_2 is unknown (as a matter of fact, the consumer may be unable to spend all his wealth in period t+1 owing to the rationing on the goods markets (<u>forced savings</u>)). If ρ_i is a continuous function, this maximum exists. Now let N_i be a may be empty subset of $K \cup H_i$ describing the markets on which our consumer receives a quantitative message, represented by $\xi_i \in R^{N_i}$. The vector ξ_i represents constraints on the consumer's net trades. By assumption, $\xi_n \geq 0$ if $n \in N_i \cap K$ and $\xi_n \leq 0$ if $n \in N_i \cap H_i$. Then, the consumer will choose $a = (x,m)$ in A_i so as to maximize $v_i(a)$ subject to $s^*.a = s^*.e_i(t)$, $x_n \leq \xi_n$ for all $n \in N_i \cap K$, and $x_n \geq \xi_n$ for all $n \in N_i \cap H_i$. The set of optimal actions is denoted $\alpha_i(\xi_i, N_i)$.

We next give the definition of an equilibrium. As in the study of the neoclassical model, we shall assume that an agent receives a quantitative signal on a given market only if he is rationed on that market. Furthermore, we shall require that either supply or demand is rationed, but not both.[21] Formally, given s*, a <u>Keynesian equilibrium</u> will be defined by the actions a_j, a_i the (may be empty) sets $N_j \subset N$ and $N_i \subset K \cup H_i$, and the signals $\xi_j \in R^{N_j}$, $\xi_i \in R^{N_i}$ $(i \in I,\ j \in J)$ such that :

(E.1) $\sum_j (a_j - e_j(t)) + \sum_i (a_i - e_i(t)) = 0.$

(E.2) $a_i \in \alpha_i(\xi_i, N_i)$ <u>and</u> $a_j \in \alpha_j(\xi_j, N_j)$ <u>for all</u> i <u>and</u> j.

(E.3) ξ_i (<u>resp.</u> ξ_j) <u>is the projection of</u> $a_i - e_i(t)$ (<u>resp.</u> $a_j - e_j(t)$) <u>on</u> R^{N_i} (<u>resp.</u> R^{N_j}) <u>for all</u> i <u>and</u> j.

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(E.4) $h \in N_i \cap H_i$ <u>for some</u> $i \in I$ <u>implies</u> $h \notin N_j$ <u>for all</u> $j \in J$. <u>Further</u>, $k \in N_j \cap K_j$ <u>for some</u> $j \in J$ <u>implies</u> $k \notin N_i$ <u>for all</u> $i \in I$ <u>and</u> $k \notin N_{j'}$, <u>for all</u> $j' \in J$, $j' \neq j$.

<u>Remarks</u>. As before, the foregoing concept does not specify how shortages are distributed among agents. One can, as in the case of the neoclassical model, impose further constraints on the rationing scheme similar to (E.5), (E.6) or (E.7) (see section 2.1), and prove the existence of a Keynesian equilibrium satisfying one of these conditions. Details are left to the readers.

Finally, one can, as in the previous model, assume that an agent's expectations are influenced by perceived quantitative signals provided that one requires that such signals are indeed perceived on every market. That is, one would require $N_i = K \cup H_i$ and $N_j = N$. Then, in the foregoing definition of an equilibrium, (E.3) and (E.4) would be replaced by :

(E.3 <u>bis</u>) $\xi_{ih} = \ell_{ih}$ <u>for some</u> $i \in I$ <u>and</u> $h \in H_i$ <u>implies</u> $\xi_{jh} > \ell_{jh}$ <u>for all</u> $j \in J$. <u>Further</u>, $\xi_j = q_{jk} - q_{jk}(t-1)$ <u>for some</u> $j \in J$ <u>and</u> $k \in K_j$ <u>implies</u> $\xi_{ik} > q_{ik}$ <u>for all</u> $i \in I$ <u>and</u> $\xi_{j'k} > q_{j'k}$ <u>for all</u> $j' \in J$, $j' \neq j$.

The analysis below then applies with straightforward changes if expectations are assumed to depend continuously on the quantitative signals $\xi_j \in R^N$ and $\xi_i \in R^{K \cup H_i}$.

In order to complete the model, we have to make precise the determination of prices by the agents at the begining of period t. This can be achieved in many ways. In what follows, we give an example of how this can be done. We focus the attention on the price making behaviour of the producer j. For instance, the producer may choose the following "myopic" rule. At the outset of period t, he would forecast, in function of his knowledge of the past history of the economy, the maximum receipt that he can expect to get from the sale of $q \in R_+^{K_j}$ at date t, say $\rho_j(q)$. To each q would then

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correspond a set of optimal prices $\pi_j(q)$. Then, the producer would choose a quantity q (and therefore, a set of prices in $\pi_j(q)$) that would maximize $\rho_j(q)$ subject to $0 \leq q \leq q_j(t-1)$. A more realistic approach would be to assume that the producer chooses a set of prices to be quoted at date t as well an <u>ex ante</u> production plan in function of his expectations regarding the states of the market at date t <u>and</u> t+1. It is not difficult (but lengthy) to write the problem that should be solved by the producer in that case. We do not go further, for this would not add much to the understanding of the short run workings of the model.

3.2. <u>An Existence Theorem</u>.

We must study the logical consistency of the model.

THEOREM 2. <u>Assume for all</u> i <u>and</u> j,

(1) $(q_j(t-1), 0) \in \text{Proj } T_j$.

(2) <u>The set</u> $Q_j = \{q \in R_+^{K_j} \mid \rho_j(q) > 0\}$ <u>is convex, and the restriction of</u> ρ_j <u>to</u> $\bar{Q}_j$ <u>is continuous and concave</u>. <u>If</u> Q_j <u>is non-empty, then for every</u> $q^* \in R_+^{K_j}$, $q^* \neq 0$, <u>there exists</u> $q \in Q_j$ <u>such that</u> $q \leq q^*$.

(3) <u>The function</u> ρ_i <u>is continuous and concave on</u> L_i.

<u>Then, there exists a Keynesian equilibrium</u>.

If one looks at the problem defining the producers'behaviour, one finds that a firm which is rationed on all markets may be forced to keep its stocks of goods, while being unable to use as inputs the goods of other producers or labor services. We must accordingly assume that the firm can pursue its activities in such a situation. This is done in condition (1), which contains as particular cases the assumptions of "free disposal", or of costless storage. The assumptions (2) and (3) are there only to guarantee nice continuity and convexity properties of the agents'demand correspondences.

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The foregoing result establishes the existence of Keynesian equilibrium for any given price system $s^* = (p^*, w^*, 1)$ quoted by the agents at the outset of period t. Given the price-making behaviour of the agents, this price system is endogeneous and is entirely determined by the past history of the economy. But imagine for a moment that we can take s^* as a variable parameter. We can then ask a question that was at the center of the controversy between classical and keynesian economists. Does there exists a choice of s^* such that, at the associated Keynesian equilibrium, all markets are cleared in the classical sense, that is, without rationing ? In order to give an answer to that query, we must recognize the fact that individual expectations about the state of the market at date t+1 are function of the prices quoted by the other agents at date t. By analogy with our study of the neoclassical model, it is intuitively clear that, if individual expectations depend "too much" on the prices that are quoted by the other agents, there may be no choice of s^* that would permit to clear the markets without rationing, as some Keynesian economists conjectured. The important fact to notice is that we need not worry about that to ensure the logical consistency of the Keynesian model as it is formulated here.

We can go further. Assume that there exists a choice of s^* such that all markets clear without rationing. Can we be sure that the corresponding wages w^* are positive ? It can be checked that, even when the marginal physical productivity of labor is positive on the set of feasible allocations, there are cases where clearing of all markets without rationing involves zero wages. It is due to the fact that, in this model, the amount of labor services demanded by firms is strongly influenced by their expectations about the future effective demand for their products. Look at the sets Q_j that are defined in (2) of Theorem 2. To simplify the exposition, assume that they are independant of the current price system s^*. It is natural to assume that Q_j is a bounded set of every j. Under reasonable assumptions on the technology T_j , this condition sets an upper bound to the amount of labor demanded by the firm at all prices and wages. Assume on the other hand that there is no desutility of labor so that the (unconstrained) labor supply is constant for all positive wages. It is then clear that, when

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the firm's expectations are pessimistic (i.e., all points of Q_j are close enough to the origin of $R_+^{K_j}$), there will be unemployment at all positive wages.[22]

Finally, we wish to remind the reader that, even if there exists a choice of s* such that equilibrium is achieved without rationing, there is no reasonable ground to claim that this price system is better than another, for the decisions taken by the agents at date t may be based upon wrong expectations about the future course of the economy.

4. CONCLUSIONS.

The foregoing analysis suggests that models using the fixed price method are better tools to describe the workings of modern economics. The basic axiom undelying neoclassical models is that prices move instantaneously to match supply and demand. In order to rationalize this postulate, economists have introduced a fictitious auctioneer who would adjust prices in function of excess demand on every market. It is hard to find markets which actually function in that way. On the other hand, in fixed prices models, a short run equilibrium is reached through adjustment on quantities. We have emphasized the fact that, in order to close such models in a consistent way, one must admit that prices are set by some agents belonging to the economic system and specify the price making behaviour of these agents. In other words, the logical consistency of the model requires the introduction of imperfect competition. This couple of assumptions (imperfect competition, plus short run adjustment on quantities) leads to a model which seems much more appropriate to describe the formation of prices which takes place in our economies.[23]

In order to make precise our fixed prices model, we assumed that prices were set by sellers. It is clear that this assumption is quite arbitrary. Indeed, the central question to be answered in subsequent studies of keynesian models seems to be : how are fixed the prices ? It is a difficult problem. It is clear at the outset, however, that any satisfactory answer to that problem should take explicitly into account

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such elements as information costs, transaction costs, and perhaps more importantly the costs involved in price quotation. Moreover, the relative sizes of the participants in each market should play a key role in the analysis, this being due to the cost of making coalitions together with the indivisibility of information.

An example may clarify this point. Consider a "big" seller facing a continuum of small buyers. Assume that these buyers must act individually (i.e., they cannot form syndicates). Assume, on the other hand, that the seller has no information about the identity of buyers.[24] Two extreme organizations of price setting can be considered in this set up. First, the seller can quote a single price independent of the buyer. This unique signal then looks like a public good and is received by every buyer. On the other hand, one can imagine that every buyer sends a signal (a price) to the seller. If price quotation involves some costs, as it should be, it is clear that the first kind of organization should prevail since it is less costly than the second one. This heuristic argument can be extended to the case of a few big sellers facing a continuum of buyers. Of course it is reversed in the case of a big buyer facing a large number of sellers. In such cases, it seems natural to assume that the "big side" of the market sets the prices. Then the fixed price method seems quite appropriate.

The method is less applicable when there are only a few participants. In this case, the costs of communication are relatively small. The buyers and the sellers will directly conclude contracts, setting at the same time the exchanged quantity and the price of exchange. The fixed price method cannot deal with these cases which should be analysed by using the methods of the theory of games.

5. PROOF OF THE RESULTS OF SECTION 2.

Proof of Theorem 1.

First one can easily show, using standard techniques, that the function v_i is well defined and continuous on $A_i \times S^*$. Furthermore, for any given $s \in S^*$, $v_i(.,s)$ is semi-strictly quasi concave with respect to $a = (q, \ell, m) \in A_i$, strictly increasing in q and m and non decreasing in ℓ. These remarks allow us to change the problem and to apply techniques similar to J. Dreze (1973 b). For any $s \in S^*$ and $\xi \in R^H_-$, let $\hat{\alpha}_i(s, \xi)$ be the set of the actions $a = (q, \ell, m) \in A_i$ which maximize $v_i(a, s)$ subject to the constraints $s.a \leq s.e_i(t)$ and $\ell \geq \xi$. From the properties of v_i, we know that $a \in \hat{\alpha}_i(s, \xi)$ implies $s.a = s.e_i(t)$. On the other hand, $[a = (q, \ell, m)$, $\hat{\alpha}_i(s, \xi)$ and $\ell_h > \xi_h$ for some h] implies $[a \in \hat{\alpha}_i(s,\xi')$ for any ξ' such that $\xi'_h < \xi_h$ and $\xi'_{h'} = \xi_{h'}$ for $h' \neq h]$. It follows that we can define in an equivalent manner an equilibrium by a price system $s = (p,w,1) \in S^*$, the actions (a_i) and (a_j), and the vectors $\xi_i \in R^H_-$ $(i \in I, j \in J)$ which satisfy (E.1) and

(E.2*) $a_j \in \alpha_j(s)$ <u>for all</u> j <u>and</u> $a_i \in \hat{\alpha}_i(s,\xi_i)$ <u>for all</u> i.

(E.3*) $w_h > w^*_h$ or $w^*_h = 0$ <u>implies</u> $\xi_{ih} < \ell_{ih}$ <u>for all i.</u>

One checks easily :

(5.1) <u>The correspondence</u> $\hat{\alpha}_i$ <u>is non-empty-, compact-, convex-valued and is u.h.c.</u>[25] <u>on</u> $S^* \times R^H_-$.

We must now study the behaviour of $\hat{\alpha}_i$ when some prices tend to zero or infinity. This is done in the next propositions.

(5.2) <u>Let</u> $(s^r, (\xi^r_i)) \in S^* \times (R^H_-)^I$ <u>be a sequence, such that</u> (ξ^r_i) <u>tends to</u> (ξ_i). <u>Consider</u>

(i) s^r <u>tends to</u> $s = (p,w,1)$ <u>such that</u> $p_k = 0$, <u>for at least one</u> k <u>in</u> K.

(ii) $\|s^r\|$ <u>tends to infinity</u>, $s^r/\|s^r\|$ <u>tends to</u> $(\bar{p},\bar{w},0)$, <u>and there exists one</u> $h \in H$ <u>such that</u> $\bar{w}_h \neq 0$ <u>and</u> $\xi_{ih} \neq 0$ <u>for all i</u>.

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If (i) or (ii) is satisfied, then for any sequence $a^r \in \sum_i \hat{\alpha}_i (s^r, \xi^r_i)$, we have $\lim \|a^r\| = +\infty$.

To prove this result, it is sufficient to show that it holds for one consumer i such that $m_i(t-1) > 0$ on the first case, and such that $\ell^*_{ih} < 0$ in the second (there always exists such a consumer). This can easily be proved, using the techniques of temporary equilibrium analysis (see Grandmont (1971)). The details are left to the reader.

The above result covers the case where the price system tends in norm towards infinity, but where the relative wages do not tend altogether towards zero. Otherwise, we have to consider the producers'demand to get a similar result.

First one can check easily, using standard arguments :

(5.3) The correspondence $\alpha_j : S^* \to A_j$ is non-empty, compact, convex valued, and u.h.c. on the set $\{s = (p,w,1) \in S^* \mid w >> 0\}$. Moreover, α_j has a closed graph.

Then :

(5.4) Let $s^r \in S^*$ be a sequence such that $\|s^r\|$ tends to infinity and $\lim (s^r/\|s^r\|) = (\bar{p},\bar{w},0)$, with $\bar{w} = 0$. Then for any sequence $a^r \in \sum_j \alpha_j(s^r)$, one has $\lim \|a^r\| = +\infty$.

Proof. If (5.4) were not true, one could find such a sequence s^r and sequences $a^r_j \in \alpha_j(s^r)$ which converge, say, to $\bar{a}_j$. Consider a j such that $\bar{p}.q_j(t-1) > 0$. We set $\bar{a}_j = (\bar{q}_1, \bar{\ell}, \bar{m}_1)$, $a^r_j = (q^r_1, \ell^r, m^r_1)$, and the corresponding production plan $(q^r_1, \ell^r, q^r_2) \in T_j$. For any j, one has $s^r.a^r_j = s^r.e_j(t)$, and so, by continuity, $\bar{p}.\bar{q}_1 = \bar{p}.q_j(t-1)$ which is strictly positive. On the other hand $0 \in T_j$ implies that, for any j, $\psi_j(s^r).q^r_2 - (p^r.q^r_1 + w^r.\ell^r) \geq 0$. Dividing this inequality by $\|s^r\|$ and going to the limit, we get $\bar{p}.\bar{q}_1 + \bar{w}.\bar{\ell} = \bar{p}.\bar{q}_1 \leq 0$, which leads to a contradiction.

Q.E.D.

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We can now come to the proof of the existence of an equilibrium with rationing. The central idea of the proof is borrowed from J. Dreze (1973 b).

Choose $\varepsilon > 0$, and define :

$$S_\varepsilon = \{\sigma = (p,w,1, \in R^{N+1} \mid p >> 0, \quad w_h^* \geq \text{Max } (0,w_h-\varepsilon) \text{ for all } h\}.$$

Therefore S_ε is a set containing S^*. For any $\sigma = (p,w,1)$ in S_ε, let us define $s = f(\sigma) \in S^*$ by $f(\sigma) = (p, \text{Max } (w,w^*), 1)$. Let $\xi^* \in R_-^H$ such that $\xi^* << \ell_i^*$, and for every i, let x_i be a continuous function defined on S_ε taking its values in R_-^H such that $x_{ih}(\sigma) = \xi_{ih}^*$ for all σ when $i \notin I_h$ (i.e. $\ell_{ih}^* = 0$) or $w_h^* = 0$, and such that :

$$x_{ih}(\sigma) = \begin{cases} \xi_h^* & \text{if } w_h \geq w_h^* , \\ 0 & \text{if } w_h = w_h^* - \varepsilon. \end{cases}$$

when $i \in I_h$ and $w_h^* > 0$.

Let us define the correspondence $\zeta : S_\varepsilon \to R^{N+1}$ by :

$$\zeta(\sigma) = \textstyle\sum_i \; (\hat{\alpha}_i \; (f(\sigma), \; x_i(\sigma)) - \{e_i(t)\}) + \sum_j \; (\alpha_j(f(\sigma)) - \{e_j(t)\}).$$

By Walras'law, we know that $f(\sigma).\zeta(\sigma) = 0$. The functions x_i being given, it is clear that any vector σ in S_ε such that $0 \in \zeta(\sigma)$ defines an equilibrium with rationing. Conversely, any equilibrium satisfying (E.1) (E.2) (E.3) (E.4) can be represented by a vector $\sigma \in S_\varepsilon$ such that $0 \in \zeta(\sigma)$ provided that the system of functions (x_i) is choosen in an appropriate way (if $w_h^* = 0$, there is no rationing since $\xi_h^* < \ell_{ih}^*$ for all i).

We shall prove the existence of such a $\sigma \in S_\varepsilon$ by adapting standard methods (Debreu (1956, 1959)). Let $\delta^r > 0$ be an increasing sequence of real numbers such that $\lim_r \delta^r = +\infty$ and, for all h, $\delta^1 > w_h^*$. Consider the sequence of compact convex sets :

$S^r = \{\sigma \in S_\varepsilon \, 1/\delta^r \leq p_k \leq \delta^r$ for all $k \in K$, $w_h^*-\varepsilon \leq w_h \leq \delta^r$ for all h such that $w_h > 0$, and $(1/\delta^r) \leq w_h \leq \delta^r$ otherwise$\}$.

From the construction of S^r, the restriction of ζ to S^r is non-empty-, compact-, convex-valued and u.h.c. Thus, for a fixed r, the image of S^r

by ζ is contained in a compact, convex subset Z^r of R^{N+1}. For any $z \in Z^r$, let

$$\mu_r(z) = \{\sigma^* \in S^r \mid \sigma^*.z \geqslant \sigma.z \text{ for all } \sigma \in S^r\}.$$

To any $(\sigma,z) \in S^r \times Z^r$, let us associate the set $\mu_r(z) \times \zeta(\sigma)$. According to the Kakutani theorem, the so defined correspondence has a fixed point (σ^r, z^r), i.e., there exists $\sigma^r = (p^r, w^r, 1) \in S^r$ and $z^r \in Z^r$ such that

$$\sigma^r.z^r \geqslant \sigma.z^r \quad \text{for any } \sigma \in S^r$$

and $\quad z^r \in \zeta(\sigma^r)$.

Let $z^r = (q^r, \ell^r, m^r)$. We first remark that $\ell^r_h \geqslant 0$ for every h such that $w^*_h \neq 0$. For if $\ell_h < 0$ for such an h, one would have $w^r_h = w^*_h - \varepsilon$, hence $x_{ih}(\sigma^r) = 0$ for all $i \in I_h$, in which case $\ell^r_h \geqslant 0$. We next wish to show that this implies $\sigma^r.z^r = 0$. If $w^*_h > 0$ and $\ell^r_h > 0$, we have $w^r_h = \delta^r$, which is greaterthan w^*_h, and, therefore, $f_h(\sigma^r) = \sigma^r_h$. It follows that $\sigma^r.z^r = f(\sigma^r).z^r$ which is equal to zero by Walras'law.

Therefore the sequence z^r is bounded, since it is bounded from below and $\sigma^1.z^r \leqslant 0$ for all r with $\sigma^1 \in S^1$. We can suppose without loss of generality that the sequence z^r converges towards $\bar{z} = (\bar{q}, \bar{\ell}, \bar{m})$.

The sequence σ^r is certainly bounded ; otherwise one could contradict (5.2) (ii) or (5.4) (if the sequence σ^r is not bounded, the sequence $f(\sigma^r)$ is also certainly unbounded). Therefore we can also suppose that the sequence σ^r converges towards $\bar{\sigma} = (\bar{p}, \bar{w}, 1) \in \bar{S}_\varepsilon$. We certainly have $\bar{\sigma} \in S_\varepsilon$, i.e. $\bar{p} \gg 0$; otherwise one could contradict (5.2) (i). Hence by continuity $\bar{z} = (\bar{q}, \bar{\ell}, \bar{m}) \in \zeta(\bar{\sigma})$ and

$$(*) \qquad 0 = \bar{\sigma}.\bar{z} \geqslant \sigma.\bar{z} \quad \text{for all } \sigma \text{ in } S_\varepsilon.$$

Now, $\bar{p} \gg 0$ implies $\bar{q} = 0$. Next, $(*)$ implies $\bar{\ell} \leqslant 0$ since $\bar{\sigma}$ is finite. Consider an h such that $w^*_h > 0$. We know by continuity that $\bar{\ell}_h \geqslant 0$. Thus $\bar{\ell}_h = 0$ when $w^*_h \neq 0$. Consider next the case $w^*_h = 0$ and $\bar{\ell}_h < 0$. That means that there is an excess supply of labor h. But we have assumed that the workers' utility functions were non decreasing with respect to labor

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services. Thus we are sure that the point $(0,0,\bar{m})$ belongs to $\zeta(\bar{\sigma})$. Finally $\bar{m} = 0$ since $\bar{\sigma}.\bar{z} = 0$. Therefore we have found a vector $\bar{\sigma} \in S_\varepsilon$ such that $0 \in \zeta(\bar{\sigma})$. This completes the proof of theorem 1.

Q.E.D.

Proof of the Corollary.

We now come to the proof of the corollary. We have seen that the rationing process was connected with the choice of the system of functions (x_i). Therefore we will show that there exists a choice of the (x_i) such that the equilibrium obtained in the above proof satisfy one of the properties (E.5), (E.6) or (E.7).

a) As for (E.5), it is sufficient to require that, given h such that $w_h^* \neq 0$, the functions $x_{ih}(\sigma)$ be equal for all $i \in I_h$.

b) To satisfy (E.6), given h such that $w_h^* \neq 0$, let us consider a consumer $i \in I_h$ and suppose that his rank on the market h is r. We impose on x_{ih} the following extra conditions. For any $\sigma \in S_\varepsilon$

$$x_{ih}(\sigma) = \begin{cases} \xi_h^* & \text{if } w_h^* \geq w_h \geq w_h^* - (\varepsilon(r-1)/|I_h|), \\ 0 & \text{if } w_h^* - (\varepsilon r/|I_h|) \geq w_h^* \geq w_h - \varepsilon. \end{cases}$$

c) To find an equilibrium which satisfies (E.7), we have to change a bit more deeply the above analysis. We assume that the consumers' utility functions are strictly quasi-concave. First for any $s \in S^*$ and $\xi \in R_-^H$, and for a given h, let us consider the (unique) action $\tilde{\alpha}_{ih}(s,\xi)$ which maximizes $v_i(a,s)$ subject to the constraints $a \in A_i$, $s.a \leq s.e_i(t)$ and $\ell_{h'} \geq \xi_{ih'}$, $h' \neq h$. This defines the effective supply on the labor market h, $\lambda_{ih}(s,\xi)$, as the component ℓ_h of the action $\tilde{\alpha}_{ih}(s,\xi)$. When this operation is repeated for all $h \in H$, we get the effective labor supply $\lambda_i(s,\xi) = (\lambda_{ih}(s,\xi))$, a point of L_i. We must prove the existence of a price system $s \in S^*$, actions (a_i) and (a_j), and vectors $\xi_i \in R_-^H$ such that (E.1) (E.2*) (E.3*) are satisfied, as well as :

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(E.7*) For all h, there exists a real number $0 \leq \beta_h \leq 1$ such that $\ell_{ih} = \beta_h \tilde{\ell}_{ih}$, with $\tilde{\ell}_i = (\tilde{\ell}_{ih}) = \lambda_i(s,\xi)$, for all i in I.

To prove this, we proceed along the same lines as in the proof of Theorem 1. S_ε is defined in the same way. What is new is that we are going to make the functions (x_i) depend on the effective supply of labor. More precisely, given $\sigma = (p,w,1) \in S_\varepsilon$ and $\tilde{\ell}_i \in L_i$, we define $x_{ih}(\sigma,\tilde{\ell}_i) \in R^H_-$ as ξ^*_{ih} when $w^*_h = 0$ or $i \neq I_h$. When $w^*_h > 0$ and $i \in I_h$, we assume :

$$x_{ih}(\sigma,\tilde{\ell}_i) = \begin{cases} \xi^*_{ih} & \text{if } w_h \geq w^*_h \\ \xi^*_{ih} + (w_h - w^*_h)\,(\xi^*_{ih} - \tilde{\ell}_{ih})/(\varepsilon/2) & \text{if } w^*_h \geq w_h \geq w^*_h - (\varepsilon/2) \\ \tilde{\ell}_{ih}\,(w_h - w^*_h + \varepsilon)\,/\,(\varepsilon/2) & \text{if } w^*_h - (\varepsilon/2) \geq w_h \geq w^*_h - \varepsilon \end{cases}$$

Thus given $\tilde{\ell}_i$, $x_{ih}(\sigma,\tilde{\ell}_i)$ is a linear function of w_h on the segments $[w^*_h, w^*_h - (\varepsilon/2)]$, $[w^*_h - (\varepsilon/2), w^*_h - \varepsilon]$, and takes the value ξ^*_{ih} when $w_h = w^*_h$, $\tilde{\ell}_{ih}$ when $w_h = w^*_h - (\varepsilon/2)$, and 0 when $w_h = w^*_h - \varepsilon$.

Let $L = \Pi_i L_i$. For any $(\sigma,\tilde{\ell}) \in S_\varepsilon \times L$, with $\tilde{\ell} = (\tilde{\ell}_i)$, define : $\tilde{\zeta}(\sigma,\tilde{\ell}) = \sum_i (\hat{\alpha}_i(f(\sigma), x_i(\sigma,\tilde{\ell}_i)) - \{e_i(t)\}) + \sum_j (\alpha_j(f(\sigma)) - \{e_j(t)\})$.

It is easy to check that an equilibrium satisfying to (E.1), (E.2*), (E.3*) and (E.7*) is characterized by the vectors $\sigma \in S_\varepsilon$ and $\tilde{\ell} = (\tilde{\ell}_i) \in L$ such that $0 \in \zeta(\sigma,\tilde{\ell})$ and $\tilde{\ell}_i = \lambda_i(f(\sigma), x_i(\sigma,\tilde{\ell}_i))$ for all $i \in I$.

In order to prove the existence of such a couple $(\sigma,\tilde{\ell})$, it suffices to slightly modify the proof of Theorem 1. One considers the same sequence of compacts S^r that approximates S_ε. Then, for every $(\sigma,\tilde{\ell})$ in $S^r \times L$ and $z \in Z^r$, one associates the set $\mu_r(z) \times \{(\lambda_i(f(\sigma), x_i(\sigma,\tilde{\ell}_i))\} \times \zeta(\sigma,\tilde{\ell})$ of $S^r \times L \times Z^r$. By applying Kakutani's fixed point theorem, one gets $(\sigma^r, \tilde{\ell}^r) \in S^r \times L$ and $z^r \in \zeta(\sigma^r, \tilde{\ell}^r)$ such that $\tilde{\ell}^r_i = \lambda_i(f(\sigma^r), x_i(\sigma^r, \tilde{\ell}^r_i))$ and $\sigma^r.z^r \geq \sigma.z^r$ for every $\sigma \in S^r$. The proof ends as the proof of Theorem 1. The details are left to the reader. Q.E.D.

6. PROOF OF THEOREM 2.

Let us come now to the second model. Given s^*, for any $\xi \in R_-^{K_j} \times R_+^{N\setminus K_j}$ let $\hat{\alpha}_j(\xi)$ be the set of the actions $a = (q_1, \ell, m) \in A_j$ where A_j is the intersection of Proj $T_j \times R_+$ with the set $\{(q_1, \ell_1, m) \mid q_{1k} \leq q_{jk}(t-1\}$, which maximize $\rho_j(q) - p^*.q_1 - w^*.\ \ell$ subject to the constraints

(i') $\quad 0 \leq q \leq q_2$

(ii') $\quad s^*.a = s^*.e_j(t)$

(iii') $\quad \xi_k \leq q_{1k} - q_{jk}(t-1) \leq 0 \qquad$ for all $k \in K_j$

(iv') $\quad q_{1k} \leq \xi_k$ and $\ell_h \leq \xi_h \quad$ for all $k \in K\setminus K_j$ and $h \in H$

(v') $\quad (q_1, \ell, q_2) \in T_j$

It is clear that [$a = (q_1, \ell, m) \in \hat{\alpha}_j(\xi)$ and $\xi_k < q_{1k} - q_{jk}(t-1)$ for some k in K_j (resp. $q_{1k} < \xi_k$ for some $k \in K\setminus K_j$; resp. $\ell_h < \xi_h$ for some $h \in H$)] implies [$a \in \hat{\alpha}_j(\xi')$ for any ξ' such that $\xi'_k < \xi_k$ and $\xi'_n = \xi_n$ for $n \neq k$ (resp. $\xi_k < \xi'_k$ and $\xi_n = \xi'_n$ for $n \neq k$; resp. $\xi_h < \xi'_h$ and $\xi_n = \xi'_n$ for $n \neq h$)]. In a similar manner, given s^*, for any $\xi \in R_-^{H_i} \times R_+^K$ let $\hat{\alpha}_i(\xi)$ be the set of actions $a = (q, \ell, m) \in A_i$ which maximize $v_i(a)$ subject to the constraints $s^*.a = s.e_i(t)$, $q_k \leq \xi_k$ for all $k \in K$ and $\ell_h \geq \xi_h$ for all $h \in H_i$. It is also clear that [$a = (q_1, \ell, m) \in \hat{\alpha}_i(\xi)$ and $q_k \leq \xi_k$ for some $k \in K$ (resp. $\ell_h > \xi_h$ for some $h \in H$)] implies [$a \in \hat{\alpha}_i(\xi')$ for any ξ' such that $\xi'_k > \xi_k$ and $\xi'_n = \xi_n$ for $n \neq k$ (resp. $\xi'_h < \xi_h$ and $\xi'_n = \xi_n$ for $n \neq h$)]. It follows that we can define in an equivalent manner an equilibrium by actions (a_i) and (a_j) and vectors $\xi_i \in R_-^{H_i} \times R_+^K$, $\xi_j \in R_-^{K_j} \times R_+^{N\setminus K_j}$ $\quad (i \in I,\ j \in J)$ which satisfy (E.1) and

(E.2*) $\quad a_i \in \hat{\alpha}_i(\xi_i)$ <u>for all</u> i <u>and</u> $a_j \in \hat{\alpha}_j(\xi_j)$ <u>for all</u> j.

(E.3*) $\quad \ell_{ih} = \xi_{ih}$ <u>for some</u> $h \in H_i$ <u>implies</u> $\ell_{jh} < \xi_{jh}$ <u>for all</u> $j \in J$.

$q_{1jk} - q_{jk}(t-1) = \xi_{jk}$ <u>for some</u> $k \in K_j$ <u>implies</u> $q_{1j'k} < \xi_{j'k}$ <u>for all</u> $j' \neq j$ <u>and</u> $q_{ik} < \xi_{ik}$ <u>for all</u> i.

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(6.1) <u>The correspondence $\hat{\alpha}_j$ is non-empty-, compact-, convex-valued and u.h.c. on</u> $R_-^{K_j} \times R_+^{N\setminus K_j}$.

<u>Proof</u>. First, consider the anticipated sales q corresponding to an optimal action $a_j \in \hat{\alpha}_j(\xi)$ and an anticipated output q_2 . It is clear that $q \in \bar{Q}_j$, for either $q_2 = 0$, in which case $q = 0 \in \bar{Q}_j$, or $\bar{q}_2 \neq 0$, in which case $q \in Q_j$. One can thus add the constraint $q \in \bar{Q}_j$ in the producer's problem without loss of generality. It is then trivial to check that (6.1) holds.

Q.E.D.

One also checks easily :

(6.2) <u>The correspondence $\hat{\alpha}_i$ is non-empty-, compact-, convex-valued and u.h.c. on</u> $R_-^{H_i} \times R_+^K$.

We can now come to the proof of the existence of a Keynesian equilibrium. Let ξ^* be a vector of R_-^N such that $\xi_k^* < -q_{jk}(t-1)$ for all $k \in K_j$ $(j \in J)$, and $\xi_h^* < \ell_{ih}^*$ for all $h \in H_i$ $(i \in I)$. Let $S_\varepsilon = \{\sigma \in R^{N+1} \mid s_n^* - \varepsilon \leq \sigma_n \leq s_n^* + \varepsilon \text{ for all } n \in N\}$, for some a priori given $\varepsilon > 0$. Consider a set of continuous functions $x_i : S_\varepsilon \to R_-^{H_i} \times R_+^K$ and $x_j : S_\varepsilon \to R_-^{K_j} \times R_+^{N\setminus K_j}$ $(i \in I,\ j \in J)$ that satisfy :

- for all $h \in H_i$, $x_{ih}(\sigma) = \xi_h^*$ if $w_h \geq w_h^*$, 0 if $w_h = w_h^* - \varepsilon$;
- for all $k \in K$, $x_{ik}(\sigma) = -\xi_k^*$ if $p_k \leq p_k^*$, 0 if $p_k = p_k^* + \varepsilon$;
- for all $k \in K_j$, $x_{jk}(\sigma) = \xi_k^*$ if $p_k \geq p_k^*$, 0 if $p_k = p_k^* - \varepsilon$;
- for all $n \in N\setminus K_j$, $x_{jn}(\sigma) = -\xi_n^*$ if $\sigma_n \leq s_n^*$, 0 if $\sigma_n = s_n^* + \varepsilon$.

Let us define for every $\sigma \in S_\varepsilon$,

$$\zeta(\sigma) = \textstyle\sum_i (\hat{\alpha}_i (x_i(\sigma)) - \{e_i(t)\}) + \sum_j (\hat{\alpha}_j (x_j(\sigma)) - \{e_j(t)\}).$$

It is clear from the properties of the functions x_i and x_j that any $\bar{\sigma} \in S_\varepsilon$ such that $0 \in \zeta(\bar{\sigma})$ defines a Keynesian equilibrium.

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Conversely, any Keynesian equilibrium can be described in such a way provided that the functions x_i and x_j are appropriately chosen.

The proof of the existence of a $\bar{\sigma} \in S_\varepsilon$ such that $0 \in \zeta(\bar{\sigma})$ is straightforward. The image of S_ε by ζ is contained in a non-empty compact convex set Z. For any $z \in Z$, consider $\mu(z) = \{\sigma^* \in S_\varepsilon \mid \sigma^*.z \geq \sigma.z$ for all $\sigma \in S_\varepsilon\}$. The correspondence which associates the set $\mu(z) \times \zeta(\sigma)$ to each $(\sigma,z) \in S_\varepsilon \times Z$ has a fixed point $(\bar{\sigma},\bar{z})$, i.e., $\bar{z} \in \zeta(\bar{\sigma})$ and $\bar{\sigma}.\bar{z} \geq \sigma.\bar{z}$ for all $\sigma \in S_\varepsilon$. Now, if $\bar{z}_n > 0$ for some $n \in N$, this implies $\bar{\sigma}_n = s^*_n + \varepsilon$, in which which case $\bar{z}_n \leq 0$ by construction of the functions x_i and x_j. In a similar way, $\bar{z}_n < 0$ implies $\bar{z}_n \geq 0$. Thus, $\bar{z}_n = 0$ for every $n \in N$. By Walras'law, $0 \in \zeta(\bar{\sigma})$.

Q.E.D.

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FOOTNOTES

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1. See also Arrow-Hahn (1971, ch. 14) where the possible influence of failures is discussed.

2. Benassy and Younes assume a fixed price system, while Dreze allows for price movements. In spite of apparent differences, the equilibrium concepts used by these three authors are quite similar. The special feature of Benassy's work is to base the rationing schemes on the agents' effective demands, as in Clower (1965) or Grossman (1971). In what follows, we shall use the central idea of Dreze's proof and we shall adapt it to make it closer in spirit to that of Debreu (1956, 1959) for the case with no price rigidities.

3. Younes presents an interesting contribution to the study of the optimality properties of a Keynesian equilibrium in connection with the role of money in the exchange process.

4. These assumptions are obviously restrictive. They are discussed in section 4.

5. Of course, this does not exclude the case where unemployment is due to "excessive" wages fixed by the workers.

6. For all x, y in R^N, $x \geq y$ means $x_n \geq y_n$ for all n, $x > y$ means $x \geq y$ and $x \neq y$, while $x >> y$ means $x_n > y_n$ for all n.

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7. Assumptions of this type were used by Sondermann (1971) in a temporary competitive equilibrium framework.

8. Of course, ℓ_i^* could vary with the date. That would not add much to the present analysis.

9. That is, for every x^1 and x^2 in $X_i \times X_i$, $u_i(x^1) > u_i(x^2)$ and $0 < \beta < 1$ imply $u_i(\beta x^1 + (1-\beta) x^2) > u_i(x^2)$.

10. But that means that workers then display a monopolistic price-making behaviour. We shall see more precisely in section 3 how to take into account such a behaviour.

11. See the Remark at the end of the section.

12. This type of rationing was studied by Dreze (1973 b).

13. That is, for every x^1 and x^2 in $X_i \times X_i$, $u_i(x^1) \geq u_i(x^2)$, $x^1 \neq x^2$ and $0 < \beta < 1$ imply $u_i(\beta x^1 + (1-\beta) x^2) > u_i(x^2)$.

14. This type of rationing was considered by Grossman (1971), and generalized by Benassy (1973).

15. One can replace (2) by an assumption of substituability between labor services and inputs of goods to get the same result. Assumption (2) can be suppressed when the firms do not use goods as inputs, that is, when $(q_1, \ell_1) \in \text{Proj } T_j$ implies $q_1 = 0$, for all j.

16. For existence theorems in similar frameworks, see Arrow-Hahn (1971), Sondermann (1971), Stigum (1969, 1973).

17. For an example see Grandmont (1971).

18. This formulation covers the case of "competitive expectations", when $\rho_j(q) = \bar{p}.q$ for some fixed $\bar{p} \in R_+^{K_j}$. But this case is not very interesting.

19. We are implicitly assuming "free disposal" at date t+1.

20. Here again, the case of "competitive expectations" $\rho_i(\ell) = \bar{w}.\ell$ for some $\bar{w}$, is a particular case of the analysis.

21. This restriction is borrowed from Dreze's paper (1973 b). Of course, this restriction was not needed in section 2, since there, only the labor supply had to be rationed. For a study of such a restriction in connection with the role of money in the exchange process, see Younes (1973).

22. This argument of course depends crucially on the assumption of an inelastic labor supply, i.e., a labor supply that is bounded away from zero when money wages vary by stay positive. It must be noted that the argument no longer holds in the case of "Competitive expectations" as was shown in section 2.

23. One can notice that, if the agents have competitive expectations, and if the prices are fixed at their neoclassical equilibrium values, the fixed price model leads to the same allocation as the neoclassical one. In this respect, the keynesian model appears as a generalization of the neoclassical one.

24. This means that the cost of identification of the buyers is very high, which precludes any discriminatory tarification on the part of the seller.

25. A correspondence α from the metric space X into the metric space Y is A-valued if $\alpha(x)$ has the property A for every x in X. Further α is upper hemicontinuous (u.h.c.) if the set $\{x \in X \mid \alpha(x) \subset G\}$ is open in X for every open subset G of Y.