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DISEQUILIBRIUM EXCHANGE IN BARTER AND MONETARY ECONOMIES

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C E P R E M A P

Many studies in monetary economics present monetary exchange as a way of strictly enlarging the set of possible trades. On the other hand we are reminded by R. CLOWER (1967) (1971) that before anything else, the institution of monetary exchange is a restriction on the set of possible trades (namely, only trades involving the good "money" on one side are allowed), which may constrain considerably exchange at disequilibrium prices. In this and other issues, we cannot expect the traditional general equilibrium approach to help us, since it deals only with equilibrium trading, and does not pay attention either to the institutional framework, or to the actual functioning of a decentralized economy.

So we shall provide in this study a model allowing us to analyze the working in disequilibrium of economies with different institutional structures, ranging from barter to monetary exchange ; within this model, two main results will be achieved :

- The actual and <u>decentralized</u> working of an economy at non-equilibrium prices will be described in very different institutional settings
- The efficiency properties of fix-price equilibria for these different institutional arrangements will be compared.

I - PRESENTATION OF THE MODEL.

1) The institutional framework and the exchange relation.

Our economy will consist of n agents $(i=1,\dots,n)$ exchanging a set of ℓ goods $(h=1,\dots,\ell)$ on different markets. The institutional framework in this economy will be essentially defined by the "exchange

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relation" E, i.e. the specification of the different pairs of goods which can be traded directly against each other (1). More precisely, if we consider two different commodities h and h', we shall say h E h' if and only if there is a market, or trading post, which we shall denote by (h,h'), on which the individuals can trade directly h against h'. Let L be the number of markets, and E their set :

$$(h,h') \in E \iff h \; E \; h'$$

Different exchange relations can describe, as was shown by CLOWER (1967), completely different market structures.

For example, a <u>pure monetary economy</u> (with m being the index for money) will be defined by :

$$h \; E \; h' \iff h = m \quad \text{or} \quad h' = m \; ,$$

which means that any trade must always have the good money on one side. The pure monetary economy will thus have $L = \ell - 1$ markets.

At other end of the spectrum, the <u>barter</u> economy will be defined by :

$$h \; E \; h' \qquad \forall h \; , \; \forall h'$$

Each good can be traded against any other good ; the barter economy will thus have $L = \ell(\ell-1)/2$ trading posts.

Between these two "extremes" all intermediate cases, like "non pure monetary economies", etc... can be described within this framework. One thing we can remark, following CLOWER (1971) is that, except in the pure monetary economy, there is no such thing as a "market for good h", as is often implied in neoclassical writings, but only markets of good h against specific goods.

Also in this model <u>no restrictions a priori</u> will be put on the possibilities of exchange, other than those resulting from the "Exchange relation". In particular, any good which is traded in more than one market can serve as a medium of exchange. This means for example that

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(1) This concept was introduced by R. CLOWER (1967), to which the reader is referred for more details and examples.

our barter economy will be one of indirect barter (2).

Each agent i will visit successively all markets. On market (h,h') he will express a net demand of good h against good h'.

2) An outline of the model.

As we said we want to describe in the different institutional settings seen above the dynamic working in real time of our economy. Time will here be seen as a succession of periods, or "market days", indexed by t. At each period the agents receive an initial endowment, visit successively all markets, emitting demands and actually realizing transactions. At the end of a market day, the agents consume what they acquired through trading. In order to keep the analytics simple, we shall work with a pure flow model (3). The dynamic element of the model is provided by the accumulation of information about trade possibilities on each market, which the agents carry from one period to the other.

As we shall see the process of information accumulation and revision implies some sequentiality of decisions, more specifically that individuals visit markets sequentially. In order not to complicate the analysis, we shall assume that each individual visits all markets once each period, and in a given order (4).

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(2) The polar case, direct barter, where no good can serve a priori as a medium of exchange, has been studied by VEENDORP (1970). It represents a strong additional restriction on exchange, and thus yields quite different results.

(3) This implies for example that money will be a commodity money and cannot be stored. A stock-flow model with storable fiat money has been developed elsewhere : BENASSY (1974a). However introducing storage of goods would have made the model much too heavy for our purposes.

(4) One sees easily that the description of the dynamic process and the existence proofs can be transposed without problem in the case where each individual visits only a limited number of markets. By relabeling markets in an appropriate way, we can so describe a number of more realistic situations : for example meetings of agents two by two as in OSTROY (1973) or OSTROY-STARR (1973) can be described by characterizing a "good" by its physical characteristics and the pair of traders.

Finally, as we shall focus essentially here on quantity adjustments, prices will be assumed fixed and given (but not at their general equilibrium value) throughout the analysis (5), an approach similar to HICKS' (1965) fixprice method.

3) Final transactions and market exchanges.

As we said, we make throughout the assumption that relative prices are fixed and constant during the period of analysis at each trading post. Since we will not consider transaction costs, we assume, for simplicity, that there is a unit of account in which prices are expressed :

$$p_1, \ldots, p_h, \ldots, p_{h'}, \ldots, p_\ell$$

The basic quantity variable is the exchange carried on market (h,h') by agent i :

$\lambda^i_{hh'}$: Volume of demand of good h against good h' (expressed in units of account).

If we call $\delta_{hh'}$ the excess demand vector corresponding to the unit transaction on market (h,h'), it has coordinates $1/p_h$ for h, $-1/p_{h'}$ for h', zero for the others, and the excess demand vector corresponding to $\lambda^i_{hh'}$ is $\lambda^i_{hh'}\ \delta_{hh'} \in R^\ell$. We shall denote by $\lambda^i \in R^L$ the vector of all these exchanges.

But trades on individual markets are only intermediate quantities for an agent. What he is interested in (i.e. what appears in his utility function) is the vector of his final transactions $z_i \in R^\ell$, whose expression is :

$$z_i = \sum_{(h,h')\in E} \lambda^i_{hh'}\ \delta_{hh'}$$

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(5) Again the number of possible institutional frameworks makes difficult to treat dynamically price changes. For such a study in monetary economies, see BENASSY (1973a) (1974b), HOWITT (1974).

Examples.

In a barter economy, the above relation will write :

$$z_{ih} = \sum_{h' \neq h} \frac{\lambda^i_{hh'}}{p_h}$$

(with the evident sign rule $\lambda^i_{hh'} = - \lambda^i_{h'h}$) while in a pure monetary economy it will be :

$$\begin{cases} z_{ih} = \dfrac{\lambda^i_{hm}}{p_h} & h \neq m \\ z_{im} = - \sum\limits_{h \neq m} \dfrac{\lambda^i_{hm}}{p_m} \end{cases}$$

Remark.

As suggested by the above formulas and examples, to one net transaction vector z_i will correspond a unique exchange plan λ^i only in the case of a monetary economy. In all other cases, and notably for the barter case, the sequence of exchanges to carry in order to attain a given final transaction vector z^i is indeterminate (6).

So the problem of a typical trader is :

- to choose an ultimate transaction vector z_i maximizing his utility
- then choose a particular exchange plan λ^i yielding the above chosen z_i.

In usual general equilibrium analyses, only the ultimate transactions vectors z_i are derived, without caring much about what happens at the individual trading posts (i.e. about the λ^i's). This was possible because of two more or less implicit assumptions :

- Traders are unconstrained at all trading posts (the equilibrium approach)
- Exchanges at all trading posts are coordinated by an "auctioneer".

Evidently in a decentralized dynamic disequilibrium framework like ours, both these assumptions are unacceptable, and we shall have to take explicitly into account what happens at each particular trading post.

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(6) This, as we shall see in connection with the work of OSTROY (1973), OSTROY-STARR (1973), is particularly important in the dynamics of the model.

4) Summary of the study.

So we shall first study a market in disequilibrium, seeing how transactions are realized and how individuals perceive their trading possibilities in the exchange process (Section II).

Then we shall see how an agent expresses rationally his demand at each trading post in function of these possibilities (Section III).

The interaction of agents on all markets generates a dynamic process of quantity adjustments, leading eventually to "stable" positions or "equilibria". In section IV, the process will be described and the existence of equilibria proved. At this point, we shall provide a simple example showing how this dynamic process works and how an equilibrium is reached (Section V).

In Section VI the efficiency properties of equilibria for different institutional frameworks will be investigated ; in particular barter and monetary equilibria will be compared.

Finally, in a concluding section, we shall try to interpret these results, and to relate them to different lines of research.

II - MARKETS IN DISEQUILIBRIUM.

In this chapter, we shall consider the working of a particular market (h,h') at a given market day t (the index t will consequently be omitted). There are n traders in the economy (i=1,...,n). Trader i comes on market (h,h') with a demand for exchange $\tilde{\lambda}^{i}_{hh'}$. He will realize transactions (noted $\bar{\lambda}^{i}_{hh'}$) and perceive constraints on his exchanges (noted $\bar{\bar{\lambda}}^{id}_{hh'}$, $\bar{\bar{\lambda}}^{is}_{hh'}$). We shall now see how these are determined.

1) Rationing and actual transactions.

Consider many traders (i=1,...,n) coming on a particular market (h,h') with demands for exchange $\tilde{\lambda}^{i}_{hh'}$. Generally the aggregate excess demand is different from zero

$$\tilde{\lambda}_{hh'} = \sum_{i=1}^{n} \tilde{\lambda}^{i}_{hh'} \neq 0$$

Since actual transactions $\bar{\lambda}^{i}_{hh'}$ must sum to zero, a rationing scheme is necessary to go from effective demands $\tilde{\lambda}^{i}_{hh'}$ to actual transactions $\bar{\lambda}^{i}_{hh'}$. We shall assume :

$$\bar{\lambda}^{i}_{hh'} = F^{i}_{hh'} [\tilde{\lambda}^{1}_{hh'}, \ldots, \tilde{\lambda}^{n}_{hh'}]$$

with $\displaystyle\sum_{i=1}^{n} F^{i}_{hh'} [\tilde{\lambda}^{1}_{hh'}, \ldots, \tilde{\lambda}^{n}_{hh'}] \equiv 0$

The exact form of rationing functions depends naturally on the exchange process on market (h,h'). We shall make a number of reasonable hypotheses on these functions (7) :

- One cannot oblige an agent to transact more than he wants, or in the other direction ("voluntary exchange")

$$|\bar{\lambda}^{i}_{hh'}| \leq |\tilde{\lambda}^{i}_{hh'}| \quad \text{and} \quad \bar{\lambda}^{i}_{hh'} \cdot \tilde{\lambda}^{i}_{hh'} \geq 0$$

- Individuals on the "short" side (i.e. suppliers in case of excess demand, demanders in case of excess supply) can realize their demands

$$\tilde{\lambda}^{i}_{hh'} \cdot \tilde{\lambda}_{hh'} \leq 0 \implies \bar{\lambda}^{i}_{hh'} = \tilde{\lambda}^{i}_{hh'}$$

- Finally, we shall assume that actual transactions depend continuously on effective demands : the functions $F^{i}_{hh'}$ are continuous in their arguments.

These conditions are satisfied for a great number of rationing schemes, and real mechanisms can take many different forms, all consistent with our assumptions : rationing tickets, queueing, priority systems, proportional rationing, etc....

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(7) These conditions have been emphasized by CLOWER (1960)(1965), BARRO-GROSSMAN (1971), GROSSMAN (1971). A formulation similar to the one given here has appeared in HOWITT (1974), who gives an interesting "shopkeeper" interpretation of the decentralized functioning of each market.

2) Exchange possibilities in disequilibrium.

As we shall see in the next sections, a most important element in the exchange decision of an agent on one market is the set of trades he considers possible on the other markets (and especially future markets).

Before going further, we can make a simple remark : from the assumption of voluntary exchange, the set of trades perceived as possible on market (h,h') will always have the form :

$$\bar{\bar{\lambda}}^{is}_{hh'} \leq \lambda^{i}_{hh'} \leq \bar{\bar{\lambda}}^{id}_{hh'}$$

with $\bar{\bar{\lambda}}^{is}_{hh'} \leq 0 \leq \bar{\bar{\lambda}}^{id}_{hh'}$

Because if a transaction is possible, any transaction of the same sign and lower magnitude is also possible. $\bar{\bar{\lambda}}^{is}_{hh'}$ and $\bar{\bar{\lambda}}^{id}_{hh'}$ are constraints giving the maximum supply and demand, respectively, of good h against good h' (in units of account) that the individual thinks to be able to realize.

3) Perceived constraints on past markets.

Consider a market (h,h') on which demands $\tilde{\lambda}^{i}_{hh'}$ have been expressed and transactions $\bar{\lambda}^{i}_{hh'}$ realized. The constraints perceived during the exchange process will depend on all information available to the agents at that time, and particularly will be influenced by the demands expressed by all agents, so that we shall write :

$$\bar{\bar{\lambda}}^{is}_{hh'} = G^{is}_{hh'} [\tilde{\lambda}^{1}_{hh'},\ldots,\tilde{\lambda}^{n}_{hh'}]$$

$$\bar{\bar{\lambda}}^{id}_{hh'} = G^{id}_{hh'} [\tilde{\lambda}^{1}_{hh'},\ldots,\tilde{\lambda}^{n}_{hh'}] \quad (8)$$

We shall be essentially interested in the constraints perceived in the same direction than the demand, which we shall denote by $\bar{\bar{\lambda}}^{i}_{hh'}$.

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(8) Written under this form, we see that effective demands appear not only as desired trades, but also as signals sent by agents to the others. The fact of including all λ's in the functions does not mean that each individual knows all demands, but rather that whatever information he has may be influenced by these demands.

i.e. $\bar{\bar{\lambda}}^{i}_{hh'} = \bar{\bar{\lambda}}^{id}_{hh'}$ if $\tilde{\lambda}^{i}_{hh'} \geq 0$

$\bar{\bar{\lambda}}^{i}_{hh'} = \bar{\bar{\lambda}}^{is}_{hh'}$ if $\tilde{\lambda}^{i}_{hh'} \leq 0$

We can ask reasonably the following properties :

- If the individual has been actually constrained, it is natural to take the actual transaction as the perceived constraint, which is then objective since the agent actually experiences it :

$$|\bar{\lambda}^{i}_{hh'}| < |\tilde{\lambda}^{i}_{hh'}| \implies \bar{\bar{\lambda}}^{i}_{hh'} = \bar{\lambda}^{i}_{hh'}$$

- In the contrary, if the agent could realize his demand, he will **perceive** subjectively some possibilities for more trade in the same direction :

$$\bar{\lambda}^{i}_{hh'} = \tilde{\lambda}^{i}_{hh'} \implies (\bar{\bar{\lambda}}^{i}_{hh'} - \bar{\lambda}^{i}_{hh'}) \cdot \tilde{\lambda}^{i}_{hh'} \geq 0$$

In particular, if the agent was on the short side, he will perceive he can transact strictly more in the same direction :

$$\tilde{\lambda}^{i}_{hh'} \cdot \tilde{\lambda}_{hh'} < 0 \implies (\bar{\bar{\lambda}}^{i}_{hh'} - \bar{\lambda}^{i}_{hh'}) \cdot \tilde{\lambda}^{i}_{hh'} > 0 \ .$$

Finally, we shall assume that the functions $G^{is}_{hh'}$ and $G^{id}_{hh'}$ are continuous in their arguments ; as noted in BENASSY (1974a), the conditions under which continuity holds imply that the individual trader has more information about the market than his demand and transaction only, but are generally satisfied in decentralized trading schemes.

4) Expected constraints.

What we have said up to now evidently does not apply to expected constraints on future markets, for which no demands have yet been expressed. What would be needed here is a theory of "rational expectations". Since it does not exist for general cases, the best we can do is to have expected constraints depend upon past information, and especially past perceived constraints, as we just defined them. A simple particular case of these expectations will be considered in the description of the dynamic process.

III - EFFECTIVE DEMANDS.

1) Definition.

We now turn to the determination of the demand for exchange that a rational trader i will express on market (h,h') : following CLOWER (1965) and LEIJONHUFVUD (1968), we shall call effective demand of an individual the exchange he wishes to realize on market (h,h'), taking into account exchanges already realized and the expected constraints on future exchanges. Before giving a formalized definition, let us describe individual i : Let $\omega_i \in R_+^\ell$, $x_i \in R_+^\ell$, $z_i \in R^\ell$ be his vectors of initial endowments, consumption and net transactions. He has a utility function $U_i(\omega_i + z_i) = U_i(x_i)$ continuous and concave in its arguments.

The individuals visit all trading posts in a given order : we shall note (j,j') < (h,h') to say that market (j,j') is visited before (h,h'), (j,j') > (h,h') to say it is visited after. Our trader i has realized transactions $\bar{\lambda}^i_{jj'}$ on markets (j,j') visited before (h,h'), and expects to face constraints $\bar{\bar{\lambda}}^{is}_{jj'} \leqslant \lambda^i_{jj'} \leqslant \bar{\bar{\lambda}}^{id}_{jj'}$ on markets (j,j') he will visit afterwards (again index t is omitted).

In accordance with our definition, the effective demand on market (h,h'), $\tilde{\lambda}^i_{hh'}$ will be given by the following program :

Maximize $U_i(\omega_i + z_i)$ subject to :

$$\begin{cases} \omega_i + z_i \geqslant 0 & & & [1] \\ z_i = \sum_{(j,j')\in E} \lambda^i_{jj'}\, \delta_{jj'} & & & [2] \\ \lambda^i_{jj'} = \bar{\lambda}^i_{jj'} & (j,j') < (h,h') & (j,j') \in E & [3] \\ \bar{\bar{\lambda}}^{is}_{jj'} \leqslant \lambda^i_{jj'} \leqslant \bar{\bar{\lambda}}^{id}_{jj'} & (j,j') > (h,h') & (j,j') \in E & [4] \\ \omega^i_{jj'} \geqslant 0 & \forall\, (j,j') \in E & & [5] \end{cases}$$

with $\omega^i_{jj'} = \omega_i + \sum_{(k,k')<(j,j')} \lambda^i_{kk'}\, \delta_{kk'}$: this is the commodity

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bundle held after trading on market (j,j'). Constraints [5] simply say that at no point during the market day the trader expects to hold a negative quantity of any good (i.e. to be bankrupt). These constraints, due to the non simultaneous nature of trading, are very similar to CLOWER's (1967) well-know "expenditure constraint" (9). Constraints [3] and [4] express that, as indicated in the definition, the trader takes into account realized transactions on past markets and expected constraints on future markets.

The set of all feasible exchange patterns λ^i for agent i, given his expectations, are given by constraints [1] to [5]. We call it $\gamma^i_{hh'}(p,\bar{\lambda}^i,\bar{\bar{\lambda}}^{is},\bar{\bar{\lambda}}^{id})$. It is a subset of R^L. Among all these possible exchange patterns, the agent will choose the ones allowing him to reach the highest utility level, and announce the corresponding trade on market (h,h'). We shall call $\xi^i_{hh'}(p,\bar{\lambda}^i,\bar{\bar{\lambda}}^{is},\bar{\bar{\lambda}}^{id})$ the mapping giving these effective demands $\tilde{\lambda}^i_{hh'}$.

This mapping is obtained in a very simple way : call $V_i(\lambda^i) = U_i(\omega_i + \sum_{(h,h')\in E} \lambda^i_{hh'} \delta_{hh'})$. Consider the set of vector λ^i solving the program : Maximize $V_i(\lambda^i)$ over $\gamma^i_{hh'}(p,\bar{\lambda}^i,\bar{\bar{\lambda}}^{is},\bar{\bar{\lambda}}^{id})$. $\xi^i_{hh'}(p,\bar{\lambda}^i,\bar{\bar{\lambda}}^{is},\bar{\bar{\lambda}}^{id})$ is the projection of this set of vectors along the coordinate (h,h').

2) Remarks and properties.

a) Unicity.

We can first remark that, even if the final transaction vector the individual wants to reach is unique(which happens if his preferences are strictly convex), there will be generally many ways of obtaining it by trading on pairs of goods, and thus the correspondance giving $\tilde{\lambda}^i_{hh'}$ will be truly multivalued, so that the individual has still a choice to make among the optimum trades. Only

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(9) Indeed, if we are in a monetary economy, and if the individual makes his purchases before his sales, constraints [5] reduce to the well-known $p\, z^+_i \leqslant m_i$.

in the case of a monetary economy are the excess demand vectors and the trades on individual markets univocally associated, as we noted above. However, even if the demand correspondence is multivalued, a trader will have to announce one and only one effective demand $\tilde{\lambda}^i_{hh'}$ at a trading post. We shall see below which problems this demand selection may cause.

b) Rationality.

As we defined it the effective demand has some optimality properties :

- Any demand $\tilde{\lambda}^i_{hh'}$, in the demand correspondence is preferred or indifferent to any other demand $\lambda^i_{hh'}$, (by definition) (10).
- If the agent is constrained to trade less than his effective demand $(|\bar{\lambda}^i_{hh'}| < |\tilde{\lambda}^i_{hh'}|)$, he will prefer (or be indifferent) to exchange $\bar{\lambda}^i_{hh'}$, rather than any quantity of lower magnitude. This results simply from the convexity of $\gamma^i_{hh'}$, and the concavity of $V_i(\lambda^i)$.

IV - THE DYNAMIC TRADING PROCESS AND EQUILIBRIUM.

1) Definition of an equilibrium.

Before going to the definition, it may be useful to contrast the functioning of our economy with the Walrasian one : in the Walrasian framework, everything happens in one single period ; prices vary and agents recontract until all excess demands are zero, and trades coordinated at each trading post. Then only do transactions actually take place.

Here in the contrary at each period or market day the agents visit all markets, propose effective demands in an uncoordinated and decentralized way, as we saw above, realize actual transactions, and

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(10) We take here implicitly the "utility" of an exchange $\lambda^i_{hh'}$, to be the maximum utility $V_i(\lambda^i)$ obtainable by feasible exchange vectors $\lambda^i \in \gamma^i_{hh'}(p, \bar{\lambda}^i, \bar{\bar{\lambda}}^{is}, \bar{\bar{\lambda}}^{id})$ whose component (h,h') is $\lambda^i_{hh'}$.

consume the final outcome of their exchanges.

In this dynamic context, traditional equilibrium definitions would not make much sense, and we shall adopt the definition of an equilibrium as a self-reproducing state. As prices are fixed, responses to disadjustments between demand and supply are quantity movements. We shall thus define an equilibrium with fixed prices as a situation where quantities are "stabilized", or more precisely a set of self-reproducing effective demands.

The dynamic evolution of the system will be provided by the learning behavior of the agents, who modify at each period their expectations about future constraints in light of the constraints they have perceived. At equilibrium, since perceived constraints are also self-reproducing, agents will anticipate them correctly. So, in order to define our equilibrium, we only have to specify the dynamic process which gives effective demands in period t as function of effective demands in the preceding periods.

2) The dynamic process.

The recursive process governing the evolution of effective demands through time can now be naturally inferred from the preceding sections :

Assume effective demands $\tilde{\lambda}^{i}_{hh'}(t-1)$ have been expressed in t-1 on all markets. The agents will have perceived constraints :

$$\bar{\bar{\lambda}}^{id}_{hh'}(t-1) = G^{id}_{hh'}\,[\tilde{\lambda}^{1}_{hh'}(t-1),\ldots,\tilde{\lambda}^{n}_{hh'}(t-1)]$$

$$\bar{\bar{\lambda}}^{is}_{hh'}(t-1) = G^{is}_{hh'}\,[\tilde{\lambda}^{1}_{hh'}(t-1),\ldots,\tilde{\lambda}^{n}_{hh'}(t-1)]$$

We take the simple expectations pattern : expected constraints in t equal perceived constraints in t-1 (11) ; knowing expected constraints in t, we can deduce sequentially effective demands in t by solving the known programs :

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(11) More general expectations patterns are easily considered. See BENASSY (1973b), (1974a, appendix).

Maximize $U_i(\omega_i + z_i)$ subject to :

$$\begin{cases} \omega_i + z_i \geqslant 0 & [1] \\ z_i = \sum_{(j,j')\in E} \lambda^i_{jj'}\, \delta_{jj'} & [2] \\ \lambda^i_{jj'} = \bar{\lambda}^i_{jj'}(t) \quad (j,j') < (h,h') \quad (j,j') \in E & [3] \\ \bar{\bar{\lambda}}^{is}_{jj'}(t-1) \leqslant \lambda^i_{jj'} \leqslant \bar{\bar{\lambda}}^{id}_{jj'}(t-1) \quad (j,j') > (h,h') \quad (j,j')\in E & [4] \\ \omega^i_{jj'} \geqslant 0 \quad \forall\, (j,j') \in E & [5] \end{cases}$$

At each market, the trader selects <u>one</u> effective demand $\tilde{\lambda}^i_{hh'}(t)$ in the effective demand correspondence $\xi^i_{hh'}[p, \bar{\lambda}^i(t), \bar{\bar{\lambda}}^{is}(t-1), \bar{\bar{\lambda}}^{id}(t-1)]$ (we shall come back below on the problems this selection poses). We see that demands must be determined on all markets in their order of visit because of constraints [3].

We shall obtain in this way effective demands $\tilde{\lambda}^i_{hh'}(t)$ as functions of $\tilde{\lambda}^i_{hh'}(t-1)$. An equilibrium will simply be a fixed point of this recursive process, i.e. a set of effective demands $\tilde{\lambda}^{i*}_{hh'}$ such that :

$$\tilde{\lambda}^i_{hh'}(t-1) = \tilde{\lambda}^{i*}_{hh'} \Longrightarrow \tilde{\lambda}^i_{hh'}(t) = \tilde{\lambda}^{i*}_{hh'}$$

To these equilibrium effective demands will correspond equilibrium transactions at each trading post $\bar{\lambda}^{i*}_{hh'}$, and a final transactions vector $\bar{z}^*_i$

$$\bar{z}^*_i = \sum_{(h,h')\in E} \bar{\lambda}^{i*}_{hh'}\, \delta_{hh'}$$

3) <u>Existence of fix-price equilibria.</u>

As the preceding description of the dynamic trading process clearly shows, there will be an equilibrium if the following mapping has a fixed point :

$$\tilde{\lambda}^i_{hh'} \longrightarrow \xi^i_{hh'}\, [p, \bar{\lambda}^i, \bar{\bar{\lambda}}^{is}, \bar{\bar{\lambda}}^{id}]$$

$$\text{with} \quad \bar{\lambda}^{i}_{hh'} = F^{i}_{hh'}[\tilde{\lambda}^{1}_{hh'},\ldots,\tilde{\lambda}^{n}_{hh'}]$$

$$\bar{\bar{\lambda}}^{is}_{hh'} = G^{is}_{hh'}[\tilde{\lambda}^{1}_{hh'},\ldots,\tilde{\lambda}^{n}_{hh'}]$$

$$\bar{\bar{\lambda}}^{id}_{hh'} = G^{id}_{hh'}[\tilde{\lambda}^{1}_{hh'},\ldots,\tilde{\lambda}^{n}_{hh'}]$$

This mapping will have a fixed point if it is an uppersemicontinuous mapping with convex values from a compact convex set into itself.

a) The compact.

From their definition, effective demands are evidently bounded : $\forall\ i \quad \forall\ (h,h') \quad - p\ \omega_i \leq \tilde{\lambda}^{i}_{hh'} \leq p\ \omega_i$.

We shall take the product of these intervals as the above compact convex set.

b) Upper semi-continuity and convexity.

The set $\gamma^{i}_{hh'}\ [p,\bar{\lambda}^{i}, \bar{\bar{\lambda}}^{is}, \bar{\bar{\lambda}}^{id}]$ is convex and continuous in its arguments. As $\bar{\lambda}^{i}$, $\bar{\bar{\lambda}}^{is}$, $\bar{\bar{\lambda}}^{id}$ are themselves continuous functions of the initial demands $\tilde{\lambda}^{i}_{hh'}$ (because of the continuity of the functions $F^{i}_{hh'}$, $G^{is}_{hh'}$, $G^{id}_{hh'}$), the set $\gamma^{i}_{hh'}$ is continuous in the initial effective demands. Maximizing $V_i(\lambda^{i})$, which is concave and continuous in λ^{i}, over this set, yields a subset of R^{ℓ} which is convex and varies uppersemicontinuously with initial demands. Since $\xi^{i}_{hh'}(p,\bar{\lambda}^{i},\bar{\bar{\lambda}}^{is},\bar{\bar{\lambda}}^{id})$ is a one dimensional projection of this set, it is also convex and u.s.c.

Q.E.D.

4) Demand selection and the "OSTROY problem".

We have proved in the preceding section the existence of fix-price equilibria where trading plans of all agents are implicitly coordinated. However, since we are interested in the dynamics of the model, we have to ask ourselves whether the dynamic trading process will actually lead to this coordinated fix-price equilibrium. We shall here leave aside the traditional stability analysis, but rather focus on the

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problems posed by the possible multivaluedness of the demand correspondences at each trading post. And with respect to this problem, it is easy to see that convergence towards a fix-price equilibrium will become more and more complicated as one goes from a pure monetary to a barter economy. Indeed in the pure monetary economy, the individual, when coming to a market, has only to choose his preferred ultimate transaction. His demand on the market is then deduced by a one to one correspondence.

In the contrary in the barter economy (or a non pure-monetary economy) multiple trading plans (λ^i) are associated to one ultimate transaction, (z_i) so that the individual has still one choice to make, basically the choice of the media of exchange he will use. And it is most likely that dynamically he will make many wrong choices, ending at the end of each period with goods he does not want. This problem has been studied brilliantly by OSTROY (1973) OSTROY-STARR (1973) who show that, unless a medium of exchange is imposed institutionally, the process of finding the right media of exchange (or trading plans) implies either some centralization (ruled out in this model), or a great consumption of time (i.e. utility losses since actual trading takes place in time).

5) <u>An equilibrium property</u>.

Until now, in order to show the dynamics if the system, we have somehow privileged the analysis of the system trading post by trading post. However, in order to study the efficiency properties of fix-price equilibria, it will be useful to have a simple characterization of the vectors of transactions $\bar{z}_i^*$ at equilibrium. And indeed they verify an interesting and symmetrical property : at a fix-price equilibrium, the vector of final transactions of an agent maximizes his utility, subject to the budget, positivity and transactions constraints, and perceived constraints on all markets ; i.e. $\bar{z}_i^*$ (and $\bar{\lambda}^{i*}$) are solution of :

Maximize $U_i\ (\omega_i + z_i)$ subject to

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$$\begin{cases} \omega_i + z_i \geqslant 0 \\ z_i = \sum_{(h,h')\in E} \lambda^i_{hh'} \, \delta_{hh'} \\ \bar{\bar{\lambda}}^{is}_{hh'} \leqslant \lambda^i_{hh'} \leqslant \bar{\bar{\lambda}}^{id}_{hh'} & \forall\ (h,h') \in E \\ \omega^i_{hh'} \geqslant 0 & \forall\ (h,h') \in E \end{cases}$$

We can notice the symmetric role of perceived constraints, which is quite natural since expected constraints are correctly anticipated at a fix-price equilibrium.

Proof (Sketch) : Assume in the contrary there exists a vector $\hat{\lambda}^i$ satisfying the above constraints and such that $V_i(\hat{\lambda}^i) > V_i(\bar{\lambda}^{i*})$. Take the first component (h,h') such that $\hat{\lambda}^i_{hh'} \neq \bar{\lambda}^{i*}_{hh'}$. Clearly $\hat{\lambda}^i \in \gamma^i_{hh'}\, [p, \bar{\lambda}^i, \bar{\bar{\lambda}}^{is}, \bar{\bar{\lambda}}^{id}]$; also $\hat{\lambda}^i_{hh'} \in [\bar{\bar{\lambda}}^{is}_{hh'}, \bar{\bar{\lambda}}^{id}_{hh'}]$. But we know that $\bar{\lambda}^{i*}_{hh'}$ is preferred or indifferent to any demand $\lambda^i_{hh'} \in [\bar{\bar{\lambda}}^{is}_{hh'}, \bar{\bar{\lambda}}^{id}_{hh'}]$ which contradicts $V_i\ (\hat{\lambda}^i) > V_i(\bar{\lambda}^{i*})$

Q.E.D.

V - A SIMPLE EXAMPLE.

We shall study here a very specific example, in order to show the working of the model in time, and the convergence towards equilibrium.

1) The economy.

Let there be three goods (1,2,3) and three traders (a,b,c) whose utility functions and endowments are :

$U_a = \text{Log } x_{a1} + \text{Log } x_{a2}$ $\qquad \omega_a = (2,0,0)$

$U_b = \text{Log } x_{b2} + \text{Log } x_{b3}$ $\qquad \omega_b = (0,2,0)$

$U_c = \text{Log } x_{c3} + \text{Log } x_{c1}$ $\qquad \omega_c = (0,0,2)$

We can remark that this is a typical case of "no coïncidence of wants" [12], i.e. where direct barter would yield a perpetual no trade

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(12) For more on this, see VEENDORP (1970).

situation. In order to make things simple, we shall take the prices to be the general equilibrium ones, i.e. :

$$p = (1,1,1)$$

2) Indirect barter.

We assume here the three possible markets are open and visited in the order (1,2) , (2,3) , (3,1).

In the first round of trading, each trader will try to realize directly his desired trade, i.e. a, b, c will express respectively demands :

$$\tilde{\lambda}^a_{12} = -1 \qquad \tilde{\lambda}^b_{23} = -1 \qquad \tilde{\lambda}^c_{31} = -1$$

The result in this first round is evidently no trade. We come now to the second round or "market day". What will happen intuitively is that agent b will feel the possibility of exchanging indirectly 2 against 3, by first buying one unit of 1 on market (1,2), then selling it against one unit of 3 on market (3,1), thus using good 1 as a medium of exchange.

We shall now show more precisely in our mathematical formulation how b is led to take good 1 as a medium of exchange by showing how b's effective demand on market (1,2), $\tilde{\lambda}^b_{12}$, is computed :

- Transactions constraints.

b' holdings after transacting on each of the three markets will be respectively :

$$\begin{cases} \omega^b_{12} = (\lambda^b_{12}, 2 - \lambda^b_{12} , 0) \\ \omega^b_{23} = (\lambda^b_{12}, 2 - \lambda^b_{12} - \lambda^b_{32} , \lambda^b_{32}) \\ x_b = \omega^b_{31} = (\lambda^b_{12} - \lambda^b_{31} , 2 - \lambda^b_{12} - \lambda^b_{32} , \lambda^b_{32} + \lambda^b_{31}) \end{cases}$$

The transactions constraints :

$\omega^b_{12} \geq 0 \qquad \omega^b_{23} \geq 0 \qquad \omega^b_{31} \geq 0$ yield thus

$$\begin{cases} 0 \leq \lambda^b_{12} \leq 2 \\ 0 \leq \lambda^b_{32} \leq 2 - \lambda^b_{12} \\ -\lambda^b_{32} \leq \lambda^b_{31} \leq \lambda^b_{12} \end{cases}$$

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- <u>Perceived constraints.</u>

  . On market (2.3), b has perceived no supply or demand :

$$\bar{\bar{\lambda}}^{bs}_{23} = 0 \qquad \bar{\bar{\lambda}}^{bd}_{23} = 0$$

  . On market (3,1), b has perceived a supply of one unit of 3 against one unit of 1 (coming from trader c) :

$$\bar{\bar{\lambda}}^{bs}_{31} = 0 \qquad \bar{\bar{\lambda}}^{bd}_{31} = 1$$

The perceived constraints on markets (2.3) and (3.1) yield thus :

$$\begin{cases} \lambda^{b}_{23} = 0 \\ 0 \leqslant \lambda^{b}_{31} \leqslant 1 \end{cases}$$

The program giving b's effective demand on market (1,2) is thus

$$\text{Maximize Log } (\lambda^{b}_{12} - \lambda^{b}_{31}) + \text{Log } (2 - \lambda^{b}_{12} - \lambda^{b}_{32}) + \text{Log } (\lambda^{b}_{31} + \lambda^{b}_{32}) \quad \text{s. t.}$$

$$\begin{cases} 0 \leqslant \lambda^{b}_{31} \leqslant 1 \\ \lambda^{b}_{23} = 0 \\ \omega^{b}_{12} \geqslant 0 \qquad \omega^{b}_{23} \geqslant 0 \qquad \omega^{b}_{31} \geqslant 0 \end{cases}$$

There is one unique solution $\tilde{\lambda}^{b}_{12} = +1$ (to which are associated trades $\lambda^{b}_{23} = 0$, $\lambda^{b}_{31} = +1$) : b buys one unit of 1 from a, which he will resell to c on market (3,1), thus acting as a "middleman".

The final allocation will be

$$x_a = (1,1,0) \qquad x_b = (0,1,1) \qquad x_c = (1,0,1)$$

i;e. the general equilibrium one. Two rounds only of trading have been necessary to converge to this solution, because of the very special configuration considered here.

We can remark that transactions constraints, though determining

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unequivocally the medium of exchange (good 1), were not binding in the programs giving effective demands, so that the economy could reach here the general equilibrium allocation. This may not always be the case, as we see now.

3) Monetary exchange.

Assume now that good 1 is institutionally taken as the medium of exchange, i.e. only markets (1,2) and (1,3) are open. We shall not rework in detail the programs of effective demands determination : It is easy to see that, with good 1 as money, trader b must be the middleman, and the sequence of trades he would have to carry in order for the economy to reach the general equilibrium allocation is as precedently :

$$\lambda^b_{12} = +1 \qquad \lambda^b_{31} = +1$$

We rather want to see how transactions constraints may interfere with this desired trade sequence. Indeed assume first that the order of markets is (1,2) , (3,1). Transactions constraints are :

$$\omega^b_{12} = (\lambda^b_{12} , 2 - \lambda^b_{12} , 0) \geq 0$$

$$\omega^b_{31} = (\lambda^b_{12} - \lambda^b_{31} , 2 - \lambda^b_{12} , \lambda^b_{31}) \geq 0$$

$$\text{Yielding} \quad \begin{cases} 0 \leq \lambda^b_{12} \leq 2 \\ 0 \leq \lambda^b_{31} \leq \lambda^b_{12} \end{cases}$$

We see that $\lambda^b_{12} = +1$, $\lambda^b_{31} = +1$ are feasible with this order. However take now the order of markets to be (3,1), (1,2). Transactions constraints now write :

$$\omega^b_{31} = (-\lambda^b_{31} , 2 , \lambda^b_{31}) \geq 0$$

$$\omega^b_{12} = (\lambda^b_{12} - \lambda^b_{31} , 2 - \lambda^b_{12} , \lambda^b_{31}) \geq 0$$

$$\text{Yielding} \quad \begin{cases} \lambda^b_{31} = 0 \\ 0 \leq \lambda^b_{12} \leq 2 \end{cases}$$

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Clearly transactions constraints are now binding, and the desired sequence is unfeasible ; the equilibrium of the economy will be the "no trade" situation, since no intermediation can take place.

VI - EFFICIENCY PROPERTIES OF EQUILIBRIA.

We shall try to compare here the efficiency properties of fix-price equilibria for different trading structures (i.e exchange relations) of the economy. However we first have to modify a little the model, in order to rule out trivial efficiency statements related to the presence of transactions constraints.

1) Sequentiality, transactions constraints and optimality.

As we just saw in the example above, it is possible with the same exchange relation to reach completely different equilibrium positions according to the ordering of markets ; transactions constraints have thus a great influence on efficiency, which is a trivial and non-interesting result, since in this pure flow model, no provision is made for the building of transactions stocks (13). We would thus like to get rid of these constraints in order to obtain finer efficiency results on our equilibria.

The model could indeed have been worked out without these constraints from the beginning if we had assumed that agents extend credit (in goods) to each other within the trading period. But this would have been quite disturbing in our description of the dynamic trading process as bankruptcies due to wrong expectations would have been quite likely to occur.

However this bankruptcy problem somehow disappears if we study only equilibrium states, since expectations are by construction fulfilled. So what we shall do here is to study the equilibrium efficiency properties of a slightly modified model where the constraints

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(13) Efficiency results with respect to transactions stocks building in an equilibrium framework can be found in GRANDMONT-YOUNES (1973), NIEHANS (1974).

$\omega_{jj'} \geq 0$ are suppressed throughout. (Accordingly this modified model should be considered as a tatonnement model, in opposition to the previous non-tatonnement one).

The whole theory could be reworked in much the same way. Since we shall use it in the sequel, we shall write here the modified property of the transactions vector of each agent at equilibrium : $\bar{z}_i^*$ is solution of the following program :

$$\text{Maximize } U_i(\omega_i + z_i) \quad \text{s.t.}$$

$$\begin{cases} \omega_i + z_i \geq 0 \\ z_i = \sum_{(h,h')\in E} \lambda^i_{hh'} \, \delta_{hh'} \\ \bar{\bar{\lambda}}^{is}_{hh'} \leq \lambda^i_{hh'} \leq \bar{\bar{\lambda}}^{id}_{hh'} \qquad (h,h') \in E \end{cases}$$

2) The efficiency criterion.

The usual Pareto-optimality criterion would evidently not make much sense here, since we are working with fixed given prices. So we shall adopt a more adapted criterion for efficiency : a state will be efficient if, at the given set of prices, no trades bearing on pairs of goods can improve strictly the utility of all traders involved (14). This criterion is particularly well-suited here, since the main characteristic of different trading structures is the existence (or non-existence) of markets for pairs of goods. And actually we shall see that only one trading structure, the barter economy, is efficient with respect to this criterion. Other structures, and most notably the monetary one, will be seen to be inefficient.

Before that, let us indicate shortly under which conditions these exchanges would be possible.

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(14) This criterion, and the associated conditions on marginal utilities, are found in ARROW-HAHN (1971, ch.13,Section 3). They have been used by YOUNES in a study on the optimality of monetary exchange (1973).

An agent i will want to demand good h against good h' at the given set of prices, which we shall note $h(\mathcal{P}_i)\ h'$, if and only if :

$$\begin{cases} \dfrac{1}{p_h} \dfrac{\partial U_i}{\partial z_{ih}} - \dfrac{1}{p_{h'}} \dfrac{\partial U_i}{\partial z_{ih'}} > 0 \\ \omega_{ih'} + z_{ih'} > 0 \end{cases}$$

A chain of exchanges bearing on pairs of goods and improving the utility of all traders involved (we shall call them Pareto-improving trades, or chains) will exist if one finds goods $h_1,\dots,h_k$ and traders $i_1,\dots,i_k$ such that :

$$h_1(\mathcal{P}_{i1})\ h_2 \qquad h_2(\mathcal{P}_{i2})\ h_3, \dots, h_k(\mathcal{P}_{ik})\ h_1$$

We consider here indirect exchanges, since in a "realistic" economy the absence of "double coincidence of wants" would make unsignificant the consideration of only direct exchanges (i.e. limited to two goods and two traders). A fix-price equilibrium will be efficient if no such Pareto-improving chain of exchanges exists.

3) The efficiency of barter equilibria.

In order to see that barter equilibria are indeed efficient with respect to our criterion, let us rewrite the program giving the transactions vector at equilibrium : $\overline{z}_i^*$ is solution of :

Maximize $U_i(\omega_i + z_i)$ subject to

$$\begin{cases} \omega_i + z_i \geq 0 \\ z_{ih} = \sum\limits_{h' \neq h} \dfrac{\lambda^i_{hh'}}{p_h} & \forall\ h \\ \overline{\overline{\lambda}}^{is}_{hh'} \leq \lambda^i_{hh'} \leq \overline{\overline{\lambda}}^{id}_{hh'} & \forall\ (h,h') \end{cases}$$

The KUHN-TUCKER conditions associated with this program are :

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$$\begin{cases} \dfrac{\partial U_i}{\partial z_{ih}} \leq R_{ih} & \text{with equality if } x_{ih} > 0 \\ \dfrac{R_{ih}}{p_h} - \dfrac{R_{ih'}}{p_{h'}} = \mu^i_{hh'} \end{cases}$$

$R_{ih} \geq 0$ can be interpreted as the exchange value of good h for trader i. It is equal to the marginal utility if the agent consumes the corresponding good.

$\mu^i_{hh'}$ is an index of rationing for agent i on market (h,h') :

. $\mu^i_{hh'} > 0$ if i is constrained in his demand of h against h' ($0 \leq \overline{\lambda}^i_{hh'} < \tilde{\lambda}^i_{hh'}$)

. $\mu^i_{hh'} < 0$ if i is constrained in his supply of h against h' ($0 \geq \overline{\lambda}^i_{hh'} > \tilde{\lambda}^i_{hh'}$)

. $\mu^i_{hh'} = 0$ if i is not constrained on market (h,h') ($\overline{\lambda}^i_{hh'} = \tilde{\lambda}^i_{hh'}$).

The conditions on rationing schemes seen above imply that <u>the $\mu^i_{hh'}$ have the same sign for all agents on a market (h,h').</u>

This property will be seen to imply the optimality of barter-equilibria ; let us first relate it to our criterion : it is easily verified that :

$$h(\mathcal{P}_i)h' \Longrightarrow \mu^i_{hh'} > 0$$

So $\mu^i_{hh'}$ appears somehow as an index of the desire to demand good h against good h'. This index having the same sign for all agents on all markets, it seems intuitive that no Pareto improving trade (in our sense) can take place at equilibrium. Indeed, assume there is an indirect Pareto-improving chain of trades :

$$h_1(\mathcal{P}_{i1})h_2 \qquad h_2(\mathcal{P}_{i2})h_3 \quad \ldots\ldots \quad h_k(\mathcal{P}_{ik})h_1$$

This would imply :

$$\mu^{i1}_{h_1h_2} > 0 \quad \mu^{i2}_{h_2h_3} > 0 \; \ldots\ldots \; \mu^{ik}_{h_kh_1} > 0$$

And by the above sign property, we would have for example :

$$\mu^{i1}_{h_1h_2} + \mu^{i1}_{h_2h_3} + \ldots\ldots + \mu^{i1}_{h_kh_1} > 0$$

which is impossible, since by definition of the μ's the left hand side is identically zero.

Q.E.D.

4) Inefficiency of other trading structures.

As we saw in the preceding section, the existence of a complete set of markets in the barter structure ensures that no potential trades on pairs of goods remain unrealized. An intuitive reasoning shows us that the other structures, where some markets are missing, should be expected to be inefficient with respect to our criterion : indeed, if a market (h,h') does not exist (h $\not\in$ h'), it is "likely" that there will coexist agents who would like to demand h against h', and agents who would like to supply h against h' (In direct or indirect trades), possibility which was ruled out if the market (h,h') exists.

We can show this in a more formalized way by observing that the program giving transactions at a fix-price equilibrium can be rewritten as: $\overline{z}^*_i$ is solution of :

Maximize $U_i\ (\omega_i + z_i)$ s.t.

$$\begin{cases} \omega_i + z_i \geq 0 & \\ z_{ih} = \sum\limits_{h' \neq h} \dfrac{\lambda^i_{hh'}}{p_h} & \forall\ h \\ \lambda^i_{hh'} = 0 & \forall\ (h,h') \notin E \\ \overline{\overline{\lambda}}^{is}_{hh'} \leq \lambda^i_{hh'} \leq \overline{\overline{\lambda}}^{id}_{hh'} & \forall\ (h,h') \in E \end{cases}$$

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The KUHN-TUCKER conditions are written as previously :

$$\begin{cases} \dfrac{\partial U_i}{\partial z_{ih}} \leqslant R_{ih} \\ \dfrac{R_{ih}}{p_h} - \dfrac{R_{ih'}}{p_{h'}} = \mu^i_{hh'} \end{cases}$$

However the $\mu^i_{hh'}$ have now the same sign for all agents <u>onlyfor pairs $(h,h') \in E$</u>. For all the other pairs $(h,h') \notin E$ (i.e. the "missing" markets), the $\mu^i_{hh'}$ can have any sign, leaving thus the possibility for unrealized trades on these markets.

The possibility of finding Pareto-improving trades evidently increases when the number of "missing markets" increases. It may be particularly high in a pure monetary economy, which has the smallest number of markets.

Indeed, in a monetary economy, the only determinate relations between exchange values will be :

$$\frac{R_{ih}}{p_h} - \frac{R_{im}}{p_m} = \mu^i_{hm}$$

μ^i_{hm} having the same sign $\forall$ i. So if we consider the set of goods which are in excess demand against money (the same would apply with goods in excess supply) there is nothing which ensures us that the "exchange values" of these goods will be ranked according to the same order for all individuals, so that Pareto-improving trades will most likely be possible.

The worst situation is evidently that of generalized excess demand or supply : in this case there is no relation a priori between the exchange values of all non monetary goods, so that there will be normally a very high number of profitable and yet unrealized transactions (15).

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(15) For examples, see BENASSY (1973b) (1974a,c).

CONCLUSIONS.

A number of conclusions can be drawn from the above analysis :

First we give a completely decentralized picture of the working of an economy at disequilibrium prices : transactions are carried at each trading post in a completely autonomous way. The final "consistency" between the actions of all agents is brought by individual adjustments in real time, not by the fictitious operation of a clearing-house-auctioneer. In all this description, the specification of the institutional framework is absolutely essential.

The second main conclusion of the study pertains to the relative performances of different trading structures (we shall essentially compare the two polar cases, the barter and pure monetary economies ; intermediate cases can be handled as well).

Indeed, if we compare the "equilibrium" positions (in our sense of self-reproducing states) attained by the system, we see that the barter arrangement performs much better than the pure monetary exchange : at a barter-equilibrium, no set of individuals can, by trading directly or indirectly on pairs of goods, improve their situation (in the Pareto sense) : all information about desired exchanges is transmitted. But this "no trade" condition (ARROW-HAHN (1971)) is satisfied only for barter exchange ; in the contrary, in a monetary economy, such Pareto-improving trades are generally possible, the most well-know case being that of the "multiplier" equilibria in Keynesian theory (16). The mediation of money brings an informational failure. So is verified CLOWER's (1971) affirmation that "The very essence of the role of money in economic activity lies in the fact that it constraints rather than facilitates market exchange of other commodities in situations of widespread disequilibrium".

Though, even in the very simplified framework of our model, this conclusion should be amended, since it pertains only to "equilibrium"

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(16) See CLOWER (1965) LEIJHONHUFVUD (1968)(1973) BENASSY (1973 a,b) (1974a,c).

states ; In studying the dynamic process, we saw actually that the fact of having one institutionally given medium of exchange in the pure monetary economy greatly helped the convergence towards equilibrium. In the contrary in the barter economy (or in non pure monetary economies), the problem of choosing media of exchange is indeterminate (the "OSTROY" problem) and likely to impede seriously the convergence towards equilibrium.

Also we should emphasize that we ignored in this study many aspects which are most important in the theory of exchange, like transaction, information and search costs (17). All these features generally tend to make monetary exchange more efficient, and more likely to prevail in the long run than barter exchange. They should be included in the model for more realistic results.

Thus, to summarize, the comparison of the relative efficiencies of monetary and barter exchange structures (and, of course, of any intermediate structure) involves broadly two main classes of arguments :

- First the costs associated with transactions, information coordination, and search are usually strongly increasing with the number of trading posts. These costs exist whether one is at equilibrium or disequilibrium prices and make under normal conditions monetary exchange much more efficient than barter. These are the arguments generally studied in the literature.

- On the other hand we saw in this paper that the non existence of some markets can produce at disequilibrium prices a particular type of inefficiency, "Effective demand failures" (18). While these failures tend to become negligible as one gets close to General Equilibrium, they can lead, especially in a pure monetary economy (where there are very few markets) to quite disastrous situations.

This may explain that, while monetary exchange has imposed itself everywhere, there are cases where money performs so badly its role of medium of exchange (like in hyperinflations or great depressions, which are the cases of generalized excess demand and supply) that one observes other media of exchange (cigerettes, etc...) or forms of barter to arise spontaneously.

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(17) See for example the work of CLOWER (1969)(1970), NIEHANS (1969)(1971).

(18) LEIJONHUFVUD (1973).

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