CEPREMAP

7402

NEOKEYNESIAN DISEQUILIBRIUM THEORY IN A MONETARY ECONOMY

by

Jean-Pascal BENASSY

January 1974
Revised June 1974

This paper is based on some chapters of an unpublished Ph. D. THESIS [4] for which Gerard DEBREU and Bent HANSEN were the most patient and helpful advisers. Several people kindly read and commented this paper :
R. CLOWER, F. HAHN, S.C. KOLM, G. LAROQUE, P. MALGRANGE, Y. YOUNES.

I am greatly indebted to J.M. GRANDMONT, whose perceptive comments brought numerous changes in the exposition. I also wish to thank the editors of the Review of Economic Studies and a referee for their helpful suggestions.

All remaining errors and obscurities are evidently mine.

SUMMARY

PAGE

I - INTRODUCTION

1/ PRESENTATION

Most concepts of conventional (or neoclassical) Economics hold rigorously only in the general equilibrium state, which precludes the study of Keynesian or Marxian Economics, or as well a satisfying integration with macroeconomic theory since all of these are essentially concerned with disequilibrium states, where transactions take place at non-Walrasian prices. It is our purpose in this paper to present concepts and tools allowing to study the functioning and properties of a decentralized monetary economy (1) at disequilibrium prices, in line with the work of CLOWER [9] [10] and LEIJONHUFVUD [30] [31] (2).

First the usual concept of demand is no longer valid, as soon as we do not assume instantaneous adjustment of prices to their equilibrium values ; if economic agents behave rationally, it must be replaced by a new concept, that of effective demand, introduced by CLOWER [9] taking into account quantity constraints as well as prices (the prototype of which is the consumption function).

Secondly interaction between individuals on the different markets gives rise not only to price adjustments (as in the standard Walrasian model), but to quantity adjustments very similar to the traditional dynamic multiplier, as described by LEIJONHUFVUD [30] (The "income constrained process").

A number of authors used successfully this approach to describe some macroeconomic phenomena within the framework of simple equilibrium models (BARRO-GROSSMAN [2] [3], GLUSTOFF [15], GROSSMAN [21] [22], SOLOW-STIGLITZ [36]).

./...

(1) - Analysis of non monetary economies in disequilibrium is somewhat different, and presented in another paper (BENASSY [6]).

(2) - Concepts similar in spirit to those presented by CLOWER and LEIJONHUFVUD are also found in the work by BENT HANSEN [24] and PATINKIN [34] (Ch. 13).

However these simplified formulations could handle no more than one or two goods in disequilibrium, and we would like here to reformulate the above concepts in the usual framework of General Equilibrium analysis (3).

So in what follows we shall study a general exchange economy "A la Debreu" with money in disequilibrium (production can be introduced without problem in the analysis : BENASSY [4] [7]).

In this framework, we shall formalize the effective demand concept, define Keynesian equilibria and prove their existence. We shall also prove an important result of Keynesian analysis : at non-Walrasian prices, multiplier effects associated with the monetary structure of exchange are responsible for the "inefficiency" of many Keynesian Equilibria. Finally we shall show the very important role played by expectations in this analysis.

To keep the analysis simple, prices will be assumed fixed throughout the period of analysis : this is the "extreme Keynesian" assumption that quantities react infinitely faster than prices, found for example in HICKS' [27] "fix-price" method (4).

2/ THE INSTITUTIONAL FRAMEWORK

Our analysis will hold in a pure money economy "A la Clower" [10] where money is the sole medium of exchange : "money buys goods and goods buy money ; but goods do not buy goods". Consequently, if there are ℓ goods $(h = 1 \dots \ell)$ plus money (index m), there will be ℓ markets on which money will be exchanged against each good. This will allow us to speak of the market, or the demand, for good h, meaning the market, or the demand, of good h against money. As was noted by CLOWER [11], this assimilation of goods and markets is possible only in a monetary economy.

./...

(3) - General Equilibrium under price rigidity has been studied with different approaches by DREZE [14], GRANDMONT-LAROQUE [18], YOUNES [37] [38]. We shall see however that our tatonnement equilibrium concept has similarities with those presented by DREZE, GRANDMONT-LAROQUE.

(4) - However the same concepts and tools can be used fruitfully to study the case where prices can vary (BENASSY [4] [7]).

An individual i will visit these ℓ markets successively, and express on market h a net demand of good h against money $\tilde{z}_{ih}$.

We can now sketch the functioning of the economy in our framework : agents express demands and supplies on a particular market ; then the process of exchange takes place, in which each agent realizes a transaction (being eventually rationed) and perceives quantity constraints on his exchange. Then in function of all perceived constraints, he will express new demands on the following markets ... and so on.

So we see that the "natural" formulation of our model would be one of sequential trading. Such a non-tatonnement model is sketched in the appendix.

However in this sequential framework the exposition becomes very quickly too heavy and unaesthetic. Thus we shall study in the main body of the text a symetrized tatonnement version, where effective demands are formulated simultaneously (though separately) on the ℓ markets.

Though different in this respect, the two models have in common a number of features which distinguish them from the traditional Walrasian approach.

3/ A NON-WALRASIAN APPROACH

The first new feature, which we already mentioned above, is evidently the introduction of quantity signals into demand functions, in addition to prices. As this will be a main theme of our study, we do not insist here on it.

Another novelty comes from the fact that, in our ℓ-markets framework, demands are expressed separately on each market. Since these demands will generally not be satisfied, we see that individual rationality does not imply that an individual's expressed demands $\tilde{z}_{ih}$ satisfy his budget constraint (though his transactions must). A fortiori, WALRAS' law will not be satisfied by effective demands. That this is indeed empirically verified has been noted by many writers (e.g. KORNAI [29]).

./...

Conversely, since we want to describe "realistic" models in which transactions can actually take place, we shall require that realized transactions identically sum-up to zero on each market (5), while this is a property of the equilibrium point only in conventional models.

All these modifications are clearly an important step towards realism, and join a long tradition which through CLOWER [9] and BENT HANSEN [24] goes back to KEYNES [28], and even K. MARX [32] [33].

4/ SUMMARY OF THE STUDY

Outlining briefly what follows, we shall see successively how transactions are realized on markets in disequilibrium and how individuals perceive their trading possibilities during this process (section II). We will then indicate how an individual expresses his effective demand in function of these possibilities (section III). We shall then describe the tatonnement process, and prove the existence of equilibria for rigid prices, K-equilibria, close in essence to the traditional Keynesian equilibrium (section IV). Their properties will be studied, notably the inefficiency of "multiplier" equilibria (section V), which will be illustrated through a short numerical example (section VI). In all the study, money is assumed to have an indirect utility as a store of value. We shall provide a theory of this utility in disequilibrium, and show at the same time the important role of expectations in the analysis (section VII).

As we said, all the study is carried in terms of a tatonnement process. The appendix will describe a non-tatonnement sequential trading model (corresponding broadly to the topics treated in sections III and IV).

./...

---

(5) - Which we will write below $\sum_{i=1}^{n} \bar{z}_{ih} \equiv 0$

$\bar{z}_{ih}$: transaction of trader i on market h.

II - MARKETS IN DISEQUILIBRIUM

1/ RATIONING AND ACTUAL TRANSACTIONS

Consider a market h on which the agents have expressed demands $\tilde{z}_{ih}$ ($i = 1 \ldots n$). In general, aggregate excess demand will differ from zero :

$$\tilde{Z}_h = \sum_{i=1}^{n} \tilde{z}_{ih} \neq 0$$

On the other hand, since we wanted to be able to describe an actual exchange process, we insisted that actual transactions $\bar{z}_{ih}$ should sum up identically to zero, i.e. :

$$\sum_{i=1}^{n} \bar{z}_{ih} \equiv 0$$

A rationing scheme is thus necessary in order to go from effective demands $\tilde{z}_{ih}$ to actual transactions $\bar{z}_{ih}$. We shall assume

$$\bar{z}_{ih} = F_{ih}\left[\tilde{z}_{1h}, \ldots, \tilde{z}_{nh}\right]$$

with :

$$\sum_{i=1}^{n} F_{ih}\left[\tilde{z}_{1h}, \ldots, \tilde{z}_{nh}\right] \equiv 0$$

The exact form of rationing functions depends evidently upon the exchange process on market h. We shall make a number of reasonable hypotheses on these functions (6) :

- one cannot oblige any agent to exchange more than he wants, or in the other direction ("voluntary exchange") ;
- individuals on the "short" side (i.e. suppliers if there is excess demand, demanders if there is excess supply) can realize their demands ("frictionless market").

./...

(6) - These conditions have been emphasized by CLOWER [8] [9], BARRO-GROSSMAN [2] and particularly GROSSMAN [21].

Mathematically, the two above conditions are written respectively :

- $|\bar{z}_{ih}| \leqslant |\tilde{z}_{ih}|$ and $\bar{z}_{ih} \cdot \tilde{z}_{ih} \geqslant 0$

- $\tilde{Z}_h \cdot \tilde{z}_{ih} \leqslant 0 \Rightarrow \bar{z}_{ih} = \tilde{z}_{ih}$.

Finally, we shall also assume, which is not very restrictive, that actual transactions depend continuously on effective demands :

- All F_{ih} functions are continuous in their arguments.

These conditions are satisfied for a great number of rationing schemes, and real mechanisms can take many different forms, all consistent with our assumptions : queueing or rationing tickets on goods markets, priority systems (by seniority, skills, ... etc) on labor markets, proportional rationing on bonds and equity markets, etc...

2/ EXCHANGE POSSIBILITIES IN DISEQUILIBRIUM

As we shall se later a most important element in determining the demands of the agents will be the constraints they perceive on their exchange possibilities on the different markets. Voluntary exchange implies that on a market h the set of transactions perceived as possible will form an interval :

$$\bar{\bar{z}}^s_{ih} \leqslant z_{ih} \leqslant \bar{\bar{z}}^d_{ih} \quad \text{with} \quad \bar{\bar{z}}^s_{ih} \leqslant 0 \leqslant \bar{\bar{z}}^d_{ih}$$

Since if a transaction is perceived as possible, any transaction of the same sign and lesser magnitude must also be perceived as possible.

From now on, in all the model, in order to simplify notations, we shall make the assumption that goods are "specialized" for each individual (i.e. always supplied or demanded). We shall call D_i the set of goods demanded by i ($z_{ih} \geqslant 0$), S_i the set of goods supplied ($z_{ih} \leqslant 0$).
So we need only to specify one number $\bar{\bar{z}}_{ih}$, which is the maximum quantity that individual i perceives to be able to transact on market h.

./...

Evidently :

$$\bar{\bar{z}}_{ih} \geq 0 \qquad h \in D_i$$

$$\bar{\bar{z}}_{ih} \leq 0 \qquad h \in S_i$$

We now turn to the determination of this perceived constraint.

3/ PERCEIVED CONSTRAINTS ON A MARKET

Consider now a market h on which agents have expressed demands $\tilde{z}_{ih}$ (i = 1 ... n) and realized transactions $\bar{z}_{ih}$. During the exchange process agent i will have perceived a constraint $\bar{\bar{z}}_{ih}$ on his possible transactions. In estimating this constraint, he takes into account all information he may have ; in particular, he will be influenced by the demands expressed by other agents, and we shall write :

$$\bar{\bar{z}}_{ih} = G_{ih}\left[\tilde{z}_{1h}, \ldots, \tilde{z}_{nh}\right]$$

The fact of including all $\tilde{z}_{ih}$'s as arguments of G_{ih} does not mean that each individual knows the demands of all others, but rather that whatever information he has is a function of these demands. For example, each individual knows at least the transactions he realizes :

$$\bar{z}_{ih} = F_{ih}\left[\tilde{z}_{1h}, \ldots, \tilde{z}_{nh}\right]$$

The perceived constraint functions G_{ih} should normally have the following properties :

α) If the agent is on the long side, and actually constrained to trade less than he wanted, it is natural to take his realized transaction as the perceived constraint, since he actually experiences the constraint. In this case the perceived constraint is <u>objective</u> :

$$|\bar{z}_{ih}| < |\tilde{z}_{ih}| \implies \bar{\bar{z}}_{ih} = \bar{z}_{ih}$$

./...

β) In the contrary if the agent could fulfill his demand, he will perceive subjectively some possibilities for more trade in the same direction (7)

$$\bar{z}_{ih} = \tilde{z}_{ih} \Rightarrow (\bar{\bar{z}}_{ih} - \bar{z}_{ih}) \cdot \tilde{z}_{ih} \geqslant 0$$

and generally he will indeed perceive he can trade strictly more in the same direction.

γ) This will be the case in particular if he was on the "short" side :

$$\tilde{z}_{ih} \cdot \tilde{Z}_h < 0 \Rightarrow (\bar{\bar{z}}_{ih} - \bar{z}_{ih}) \cdot \tilde{z}_{ih} > 0$$

To these most natural properties, we shall add the hypothesis that perceived constraints vary continuously with effective demands, i.e. that the functions G_{ih} are continuous in their arguments. We can remark that this hypothesis implies that the individual has, at least when he is not constrained, an information superior to his "minimal" information (which consists in his demand $\tilde{z}_{ih}$ and his transaction $\bar{z}_{ih}$). As one can see in examples, this property will usually be verified for decentralized processes, since an unconstrained individual will actually meet other agents who will propose him exchanges of greater magnitude than his own demand (or supply).

An example

Consider a market (we drop the subscript h) with one supplier ($\tilde{z}_s < 0$) and n demanders ($\tilde{z}_i > 0$). There is for the demanders a priority system, or a queue. We take, to simplify, the priority order to be the natural ranking from 1 to n.

When demander i meets the supplier, he is faced with the supply remaining after the ones before him have expressed their demands and carried their transactions, i.e. :

$$\tilde{z}_s + \sum_{i'<i} \bar{z}_{i'} = \text{Min}\left[0 , \tilde{z}_s + \sum_{i'<i} \tilde{z}_{i'}\right]$$

./...

(7) - However, even if the perceived constraint is subjective, it should be function only of signals objectively received by the individual.

This quantity (with a change of sign because of the sign conventions) is the most natural expression for i's perceived constraint, i.e. :

$$\bar{\bar{z}}_i = \text{Max}\left[0\ ,\ -\tilde{z}_s - \sum_{i'<i} \tilde{z}_{i'}\right]$$

And his transaction will be naturally :

$$\bar{z}_i = \text{Min}\ \{\ \tilde{z}_i\ ,\ \text{Max}\left[0\ ,\ -\tilde{z}_s - \sum_{i'<i} \tilde{z}_{i'}\right]\ \}$$

On this example, as well as on the general formulation, appears most clearly the close interrelatedness between the rationing schemes (F_{ih}) and the perceived constraints (G_{ih}), which is most natural since the two are complementary aspects of the same exchange process.

4/ REMARKS ON THE "SPECIALIZATION" ASSUMPTION

While the assumption of "specialized" goods is realistic in many cases (labor, consumption goods, ...) it is easily seen that it implies mathematically quite strong assumptions on endowments, namely :

$$h \in D_i \implies \omega_{ih} = 0$$

$$h \in S_i \implies .\ \omega_{ih} > 0 \quad \text{and} \quad h \in D_j \quad j \neq i$$

. or the consumption set is bounded above by ω_{ih}

For more general cases, we would have to specify, as we noted above, two perceived constraints, 1 on demand, 1 on supply

$$\bar{\bar{z}}_{ih}^{d} = G_{ih}^{d}\left[\tilde{z}_{1h}\ ,\ \dots\ ,\ \tilde{z}_{nh}\right]$$

$$\bar{\bar{z}}_{ih}^{s} = G_{ih}^{s}\left[\tilde{z}_{1h}\ ,\ \dots\ ,\ \tilde{z}_{nh}\right]$$

The whole theory carries on without difficulty provided the two functions are continuous, and the perceived constraint which has the same sign as the effective demand possesses the properties seen above.

We now revert definitively to the "specialized" case, for its notational simplicity.

./...

III - EFFECTIVE DEMANDS

1/ DEFINITION

We can now give the expression of the demand $\tilde{z}_{ih}$ that trader i will express on market h , his effective demand for good h :

Following CLOWER [9] and LEIJONHUFVUD [30], we shall call effective demand for good h the exchange the agent wishes to realize on market h to maximize his utility, taking into account the exchanges he perceives to be able to realize on the other markets (while the neoclassical demand function implicitly assumes that the individual can realize whatever exchange he wants on the other markets).

Before giving a formalized definition, let us describe agent i :

Let $\omega_i \in R_+^\ell$, $x_i \in R_+^\ell$, $z_i \in R^\ell$ be his vectors of endowments, final consumption and net transactions respectively, $\bar{M}_i \geq 0$ his initial holdings of money, $M_i \geq 0$ the quantity of money held at the end of the planning period.

He has an utility function $U_i\left[x_i , M_i\right] = U_i\left[\omega_i + z_i , M_i\right]$ continuous and concave in its arguments (money has an indirect utility as a store of value which will be derived in the last section).

$p \in R_+^\ell$ is the price vector of non monetary goods. The price of money is 1.

Let $\bar{\bar{z}}_{ih'}$ be his perceived constraint on market h' (notice these are constraints on exchanges, and thus on flows . If there are stocks in the problem, only their increases or decreases will be constrained).

The net effective demand for good h $\tilde{z}_{ih}$ will be the component number h of the optimum vector of the following program :

Maximize $U_i\left[\omega_i + z_i , M_i\right]$ subject to

./...

$$\begin{cases} p\, z_i + M_i \leqslant \bar{M}_i & \\ \omega_i + z_i \geqslant 0 & M_i \geqslant 0 \\ z_{ih'} \leqslant \bar{\bar{z}}_{ih'} & h' \in D_i \ h' \neq h \\ z_{ih'} \geqslant \bar{\bar{z}}_{ih'} & h' \in S_i \ h' \neq h \end{cases}$$

We call this last set $\gamma_{ih} \left[p \, , \, \bar{\bar{z}}_i\right]$

So what the individual does is compute an optimal exchange plan, taking into account the constraints he perceives on the <u>other</u> markets, and to announce the corresponding exchange he desires to realize on market h.

A point we should reemphasize is that this effective demand $\tilde{z}_{ih}$ is made against money, i.e. the counterpart the agent proposes to the market is the amount of money $p_h \cdot \tilde{z}_{ih}$. None of the counterparts he really wishes to offer (i.e. the other components of the optimizing vector) is transmitted to the market.

2/ EXAMPLES

Clearly, the inclusion of quantity rationing signals in the demand functions, in addition to prices, is a good step towards realism, and as a result many well-known relations in macroeconomic theory can be given a theoretical foundation within the framework of effective demand functions, while this would be impossible in a fully neoclassical analysis. We give here two of the most well-known examples :

- The consumption function, as pointed out by CLOWER [9], is the constrained demand function of individuals who cannot succeed in selling all the labor they would like to sell ; their income becomes a binding constraint and enters as an argument of the demand for goods. Actually, not only realized income should be taken into account (as in the "naive" consumption function), but future constraints as well : Cf the life cycle theories of consumption .

./...

- The accelerator, as shown by GROSSMAN [22] is the investment demand of a firm which cannot sell its notional output (i.e. profit maximizing output). Sold output becomes a constraint (and not a choice variable), and it will enter investment demand together with price variables. Here, as shown by examples in GROSSMAN [22], the importance of forecasted constraints is crucial.

IV - NEOKEYNESIAN EQUILIBRIUM

1/ DEFINITION

As we said earlier, in our period of analysis, prices are fixed and responses to inequalities of supply and demand are quantity movements. So there will be a process of quantity adjustments, in which the agents revise their effective demands in light of the constraints they perceive. If we start from a set of effective demands ($\tilde{z}_{ih}$), they generate a set of perceived constraints $\bar{\bar{z}}_{ih}$, hence a new set of effective demands that will in general differ from the original ones. Intuitively, an equilibrium will be reached when these two sets of effective demands coincide. More formally, a K-equilibrium will be a set of effective demands $\tilde{z}_{ih}$, perceived constraints $\bar{\bar{z}}_{ih}$ and realized transactions $\bar{z}_{ih}$ such that :

- $\bar{\bar{z}}_{ih} = G_{ih} (\tilde{z}_{1h} , \ldots , \tilde{z}_{nh})$
- $\tilde{z}_{ih}$ is obtained by maximization of $U_i (\omega_i + z_i , M_i)$ over the set $\gamma_{ih} (p , \bar{\bar{z}}_i)$
- $\bar{z}_{ih} = F_{ih} (\tilde{z}_{1h} , \ldots , \tilde{z}_{nh})$ "

It is easy (and may be more intuitive) to see that a K-equilibrium can be obtained as a fixed point of the following recursive tatonnement process :

Assume at time t-1 individuals have expressed effective demands $\tilde{z}_{ih} (t - 1)$ on the different markets.

./...

From these result perceived constraints $\bar{\bar{z}}_{ih}$ (t - 1). On the basis of these perceived constraints, the individual will determine a new set of effective demands $\tilde{z}_{ih}$ (t) by the following programs :

Maximize $U_i \left[\omega_i + z_i , M_i\right]$ subject to :

$$\begin{cases} p\, z_i + M_i \leqslant \bar{M}_i & \\ \omega_i + z_i \geqslant 0 & M_i \geqslant 0 \\ z_{ih'} \leqslant \bar{\bar{z}}_{ih'} (t-1) & h' \in D_i \qquad h' \neq h \\ z_{ih'} \geqslant \bar{\bar{z}}_{ih'} (t-1) & h' \in S_i \qquad h' \neq h \end{cases}$$

2/ RATIONALITY OF THE K-EQUILIBRIUM CONCEPT

From the way our K-equilibrium has been defined, it is clear that the set of transactions $\bar{z}_{ih}$ are consistent on each market, since by construction :

$$\sum_{i=1}^{n} \bar{z}_{ih} \equiv 0 \qquad \forall\, h$$

However a question which comes to mind is whether at equilibrium realized transactions are acceptable by the traders, i.e. whether they maximize utility subject to all constraints they perceive.

And indeed, it is easy to verify (by reductio ad absurdum) that the vector of transactions $\bar{z}_i$ of a trader i maximizes his utility, subject to all constraints $\bar{\bar{z}}_{ih}$ (8), i.e. in mathematical terms :

./...

(8) - Characterized in this way, our K-equilibria are formally similar to equilibria with rationing proposed by DREZE [14] with some modifications (GRANDMONT-LAROQUE [18]) : these ones are indeed defined as a set of feasible transactions maximizing the utility of each agent under quantity constraints (analogous to the $\bar{\bar{z}}_{ih}$) such that demand and supply are not rationed at the same time. I wish to thank J.M. GRANDMONT for pointing out clearly this similarity to me.

$\bar{z}_i$ maximizes $U_i \left[\omega_i + z_i \, , M_i\right]$ subject to :

$$\begin{cases} p\, z_i + M_i \leqslant \bar{M}_i & \\ \omega_i + z_i \geqslant 0 & M_i \geqslant 0 \\ z_{ih} \leqslant \bar{\bar{z}}_{ih} & h \in D_i \\ z_{ih} \geqslant \bar{\bar{z}}_{ih} & h \in S_i \end{cases}$$

3/ THE EXISTENCE OF A K-EQUILIBRIUM

A K-equilibrium will exist if the mapping

$$\{ \tilde{z}_{ih} \; (t - 1)\} \rightarrow \{ \tilde{z}_{ih} \; (t)\}$$

is an upper semicontinuous mapping with convex values from a compact convex set into itself.

a) Determination of the compact

An agent cannot supply more of good h than he has :

$$\tilde{z}_{ih} \geqslant - \omega_{ih}$$

On the other hand, he cannot demand more than what he is able to pay :

$$\tilde{z}_{ih} \leqslant \frac{p \, . \, \omega_i + \bar{M}_i}{p_h}$$

which is finite if $p_h > 0$ for all h.

Each effective demand belongs to a closed compact interval :

$$- \omega_{ih} \leqslant \tilde{z}_{ih} \leqslant \frac{p \, . \, \omega_i + \bar{M}_i}{p_h}$$

The product of these intervals is the compact convex set we are looking for.

./...

b) Upper-hemicontinuity and convexity

The set $\gamma_{ih} \left[p \, , \, \bar{\bar{z}}_i \, (t-1) \right]$ on which the individual maximizes his utility function is convex and depends continuously upon the demand $\tilde{z}_{ih}$ (t - 1).

As the utility function is itself continuous and concave, the mapping will be u.h.c. with convex values Q.E.D.

V - EFFICIENCY PROPERTIES OF K-EQUILIBRIUM

1/ THE CRITERION

One of the most appealing features of the concept of General Equilibrium is that, under very weak assumptions, it corresponds to Pareto-Optimal states (DEBREU [12]). Here, as can be easily guessed, there is no great hope that our K-equilibria will be Pareto-optimal in the usual sense (unless the price system happens to be the General Equilibrium one).

Thus, we shall adopt a more adapted criterion for efficiency : a state will be efficient if, at the given set of prices no trades bearing on pairs of goods can improve strictly the utility of all traders involved (9) :

The intuitive reason for this criterion is evidently the comparison with an indirect barter economy, where such pairwise trades are allowed. But, even with this very enlarged notion of efficiency, we will see that K-equilibria may very well be inefficient.

Before that, let us indicate shortly under which conditions these exchanges would be possible.

./...

(9) - This criterion, and the associated conditions on marginal utilities are found in ARROW-HAHN [1] (Ch.13 , section 3). They have been used by Y. YOUNES in a study on the optimality of monetary exchange [38].

An agent i will want to demand good h against good h' at the given set of prices, which we shall note h $(\mathcal{P}_i)$ h' , if and only if :

$$\begin{cases} \dfrac{1}{p_h} \dfrac{\partial U_i}{\partial z_{ih}} - \dfrac{1}{p_{h'}} \dfrac{\partial U_i}{\partial z_{ih'}} > 0 \\ \\ \omega_{ih'} + z_{ih'} > 0 \end{cases}$$

A chain of exchanges bearing on pairs of goods and improving the utility of <u>all traders involved</u> (we shall call them Pareto improving trades, or chains), will exist if one finds goods $h_1 \ldots h_k$ and traders $i_1 \ldots i_k$ such that :

$$h_1 \ (\mathcal{P}_{i1}) \ h_2 \ , \ h_2 \ (\mathcal{P}_{i2}) \ h_3 \ , \ \ldots \ , \ h_k \ (\mathcal{P}_{ik}) \ h_1$$

We consider here indirect barter exchanges, since in a "realistic" economy, the absence of double coincidence of wants would make unsignificant the consideration of only direct barter exchanges (i.e. limited to two goods and two traders). Clearly the more disaggregated the economy, the longer will be the necessary exchange chains.

A K-equilibrium will be efficient if no such Pareto-improving chain of exchanges exists.

2/ PROPERTIES OF A K-EQUILIBRIUM

To determine the properties we look for, let us write the program giving the transactions vector $\bar{z}_i$ of an agent i : as we saw above, $\bar{z}_i$ is solution of :

Maximize $U_i \left[\omega_i + z_i \ , \ M_i\right]$ subject to

$$\begin{cases} \omega_i + z_i \geq 0 & M_i \geq 0 \\ p \, z_i + M_i \leq \bar{M}_i & \\ z_{ih} \leq \bar{\bar{z}}_{ih} & h \in D_i \\ z_{ih} \geq \bar{\bar{z}}_{ih} & h \in S_i \end{cases}$$

./...

The Kuhn-tucker conditions for this program can be written :

$$\begin{cases} \dfrac{\partial U_i}{\partial M_i} \leqslant \lambda_{im} & \text{with equality if} \quad M_i > 0 \\[2em] \dfrac{\partial U_i}{\partial z_{ih}} \leqslant \lambda_{im} p_h + \delta_{ih} & \text{with equality if} \quad x_{ih} > 0 \end{cases}$$

$\lambda_{im} \geqslant 0$ can be interpreted as the exchange value of money

δ_{ih} is an index of rationing for agent i on market h :

- $\delta_{ih} > 0$ if i is constrained on his demand of h $(0 \leqslant \bar{z}_{ih} < \tilde{z}_{ih})$
- $\delta_{ih} < 0$ if i is constrained on his supply of h $(\tilde{z}_{ih} < \bar{z}_{ih} \leqslant 0)$
- $\delta_{ih} = 0$ if i is not constrained on market h $(\bar{z}_{ih} = \tilde{z}_{ih})$

The conditions on rationing schemes seen above imply that the δ_{ih} have the same sign for all agents on a market h (by convention we take $\delta_{im} = 0$)

If we define $\mu_{ih} = \dfrac{\delta_{ih}}{p_h}$ (10)

and we use the definition in the preceding section, we see that :

$$h \; (P_i) \; h' \implies \mu_{ih} - \mu_{ih'} > 0$$

./...

(10) - This quantity (divided by λ_{im}) is similar (except for the indices) to the $\mu_{i's}$ in ARROW-HAHN.

But the sign property on the δ_{ih} implies that the quantity $\mu_{ih} - \mu_{ih'}$ will have the same sign for all agents for the following pairs of goods :

- pairs in which one good is money
- pairs in which one good is in excess demand $(\tilde{Z}_h \geq 0)$, the other in excess supply $(\tilde{Z}_h \leq 0)$.

Thus no exchange chain, direct or indirect, improving strictly the utility of exchangers, can include one of the above pairs.

But, and this is the fundamental result, the above criterion has no reason to apply for pairs of goods whose excess demands are non zero, and of the same sign. Thus if we consider sets of goods all in excess demand (or all in excess supply), there will be very likely Pareto improving exchanges. In this case (which is most likely to occur if there are many goods and prices do not clear the markets) the K-equilibrium will be inefficient.

Among these inefficient equilibria (and the associated Pareto-improving exchange chains), we can operate a classification :

Sometimes, the inefficiency comes from a bad rationing scheme which allocates inefficiently rationed goods among the rationed consumers (this occurs for example if traders in a Pareto improving chain are constrained on the two goods they desire to exchange). This case is not very interesting.

3/ INEFFICIENCY AND MULTIPLIER EFFECTS

A much more interesting case arises if, in each chain, each trader is constrained on only one of the goods he trades : this makes possible a circular transmission of disturbances (multiplication).

More specifically, we shall say that there is a multiplier effect if we can find a chain of k traders $(i_1 \ldots i_k)$ and k goods $(h_1 \ldots h_k)$ all in excess demand (or all in excess supply) such that :

$$i_1 \quad \text{is} \quad \begin{cases} \text{constrained on good } h_1 \\ \text{unconstrained on good } h_2 \end{cases}$$

$$i_2 \quad \text{is} \quad \begin{cases} \text{constrained on good } h_2 \\ \text{unconstrained on good } h_3 \end{cases}$$

. . . .

$$i_k \quad \text{is} \quad \begin{cases} \text{constrained on good } h_k \\ \text{unconstrained on good } h_1 \end{cases}$$

The corresponding state is evidently inefficient.

In this case, an initial disturbance (for example, an aggravation of disequilibrium on the first market) will be transmitted with the same sign to all markets in the chain, and will ultimately come back to the first market, launching a new wave of disturbances (we recognize mechanisms similar to the traditional multiplier or multiplier-accelerator).

Generally, many such chains will be found. Multiplier effects will be evidently observed most acutely in cases of generalized excess demand or supply, since excess demands have the same sign for all goods.

The most well-known example of these inefficient states is evidently the deflationary Keynesian case : there an increase in employment would increase both firms' profits and individuals' utilities. But, unfortunately, the market does not provide any signal for the existence of such a profitable exchange.

4/ THE CAUSE OF INEFFICIENCY

As we have seen in the preceding paragraphs, there is in the inefficiency properties of Keynesian equilibria more than the inefficiency associated with non-flexible prices. But clearly there is also an informational and signalling problem, since often transactors will fail realizing trades which are both possible and profitable to everybody.

This informational failure is clearly due to the particular nature of effective demands in a monetary economy, and specifically to the fact that desired counterparts are not transmitted (11). The dissociation of purchases and sales which money permits certainly brings increased flexibility, but does not allow to transmit all desired exchanges.

As LEIJONHUFVUD says, giving the example of Keynesian deflation :

"The workers looking for jobs ask for money, not for commodities. Their notional demand for commodities is not communicated to producers ; not being able to perceive this potential demand for their products, producers will not be willing to absorb the excess supply of labor ..." ([30], p. 90).

But the ultimate cause of inefficiency should be looked for still further, in the extreme complexity of the indirect barter exchanges which would be necessary without money in our highly specialized economies :

"The fact that there exists a potential barter bargain of goods for labor services that would be mutually agreeable to producers as a group and labor as a group is irrelevant to the motion of the system. The individual steel-producer cannot pay a newly-hired worker by handing over to him his physical product (nor will the worker try to feed his family on a ton-and-a-half of cold rolled sheet a week). The lack of any "mutual coincidence of wants" between pairs of individual employers and employees is what dictates the use of a means of payment in the first place" ([30], p. 90).

./...

(11) - In mathematical terms, only $\tilde{z}_{ih}$, the h^{th} component of each optimizing vector, is transmitted, not the remainder of the vector. This point goes back to KEYNES [28] (Cf. the beginning of chapter 16). It has been elaborated brilliantly by CLOWER [9] [11] and LEIJONHUFVUD [30] [31].

VI - AN EXAMPLE

We shall give here a very simplified example, destined to show numerically the inefficiency property of "multiplier equilibria".

1/ THE ECONOMY

It will be the simplest monetary economy with three goods (1 , 2 , 3) and two agents (A , B). Both have the same utility functions.

$$U_A = \text{Log } x_{A1} + \text{Log } x_{A2} + \text{Log } x_{A3}$$

$$U_B = \text{Log } x_{B1} + \text{Log } x_{B2} + \text{Log } x_{B3}$$

But different endowments

$$\omega_A = (2 , 0 , 1)$$

$$\omega_B = (0 , 2 , 1) \qquad (12)$$

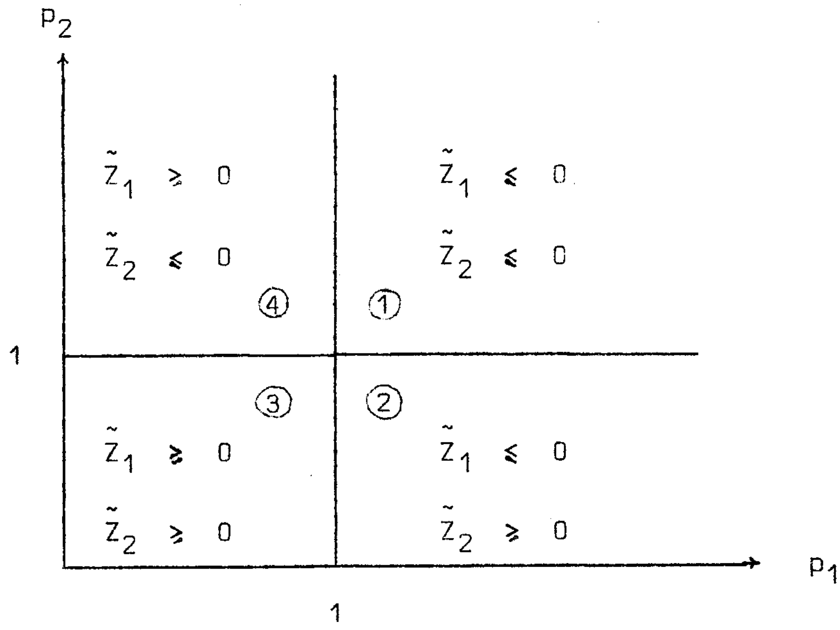
Good 3 is taken as money. Prices will be $(p_1 , p_2 , 1)$.

According to the values of p_1 , p_2 , we can distinguish 4 regions, separated by the lines $p_1 = 1$, $p_2 = 1$, according to the signs of effective demands (remark that these regions differ from the ones given by Walrasian demands). (13)

./...

(12) - Intuitively one may think of goods 1 , 2 , 3 as consumption goods, labor and money respectively. Agent A would represent aggregate firms, agent B aggregate consumers. For a more explicit treatment of firms and consumers, see BARRO-GROSSMAN [2] [3], BENASSY [4][5].

(13) - Complete calculations of transactions and excess demands (for which I acknowledge the help of P. MALGRANGE) are a bit long and have been omitted. As an example we show how to compute transactions in the region of general excess supply 1 .



From the results of section II.2, we know that exchange will be efficient in regions ② and ④, since the aggregate effective demands are of opposite sign.

In the contrary "multiplier" effects will occur in regions ① and ③. As an illustration, we shall show what happens in region ① (general excess supply).

2/ COMPUTATION OF EQUILIBRIUM TRANSACTIONS (REGION ①)

Since there is excess supply on both markets transactions will be given by the demand side, i.e. respectively :

- A's demand of good 2
- B's demand of good 1

$$\begin{cases} \tilde{z}_{A2} = \bar{z}_{A2} \equiv -\bar{z}_{B2} \\ \tilde{z}_{B1} = \bar{z}_{B1} \equiv -\bar{z}_{A1} \end{cases}$$

A's effective demand of good 2 is given by :

$$\text{Max Log } (2 + z_{A1}) + \text{Log } z_{A2} + \text{Log } (1 + z_{A3}) \quad \text{s.t.}$$

$$\begin{cases} p_1 \, z_{A1} + p_2 \, z_{A2} + z_{A3} = 0 \\ z_{A1} \geq \bar{\bar{z}}_{A1} \end{cases}$$

./...

We know that A's supply of 1 is constrained, so that the last constraint is binding, which yields :

$$p_2 \tilde{z}_{A2} = \frac{1}{2}\left[1 - p_1 \bar{\bar{z}}_{A1}\right] = \frac{1}{2}\left[1 - p_1 \bar{z}_{A1}\right]$$

We see that A's propensity to consume (out of money holdings and sales of good 1) is one half.

Symetrically, B is constrained on his sales of good 2 $(\bar{\bar{z}}_{B2} = \bar{z}_{B2})$, and his demand of good 1 is :

$$p_1 \tilde{z}_{B1} = \frac{1}{2}\left[1 - p_2 \bar{\bar{z}}_{B2}\right] = \frac{1}{2}\left[1 - p_2 \bar{z}_{B2}\right]$$

We can solve easily the above system, and obtain realized transactions :

$$- \bar{z}_{A1} \equiv \bar{z}_{B1} = \frac{1}{p_1}$$

$$- \bar{z}_{B2} \equiv \bar{z}_{A2} = \frac{1}{p_2}$$

and final holdings are :

$$A \ : \left(2 - \frac{1}{p_1} \ , \ \frac{1}{p_2} \ , \ 1 \right)$$

$$B \ : \left(\frac{1}{p_1} \ , \ 2 - \frac{1}{p_2} \ , \ 1 \right)$$

3/ INEFFICIENCY

Since aggregate excess demands are both negative in the interior of region 1, we should expect, according to the analysis of section V, Pareto improving trades bearing on goods 1 and 2 to be possible. This is easy to check by computing the propensities to exchange good 1 against good 2 at equilibrium :

$$\frac{1}{p_2}\frac{\partial U_A}{\partial z_{A2}} - \frac{1}{p_1}\frac{\partial U_A}{\partial z_{A1}} = \frac{2\,(p_1 - 1)}{2\,p_1 - 1} > 0$$

$$\frac{1}{p_1}\frac{\partial U_B}{\partial z_{B1}} - \frac{1}{p_2}\frac{\partial U_B}{\partial z_{B2}} = \frac{2\,(p_2 - 1)}{2\,p_2 - 1} > 0$$

We see that A and B would both gain in exchanging 1 against 2 directly. However, with good 3 as money, there is no way they can communicate to each other these desires for exchange.

Clearly, the opening of market (1 : 2) would restore efficiency. This can be checked without computation on the "diagonal" $(p_1 = p_2 = p)$, where A and B would exchange directly one unit of 1 against one unit of 2, reaching in this way the "General Equilibrium" allocation :

$$x_A = (1\ ,\ 1\ ,\ 1) \qquad\qquad x_B = (1\ ,\ 1\ ,\ 1)$$

A little note before leaving this example : in this case, a direct barter exchange was enough to restore efficiency. Clearly this is due to the highly aggregated character of our example's economy. In general, much more indirect trades would be necessary as was noted by LEIJONHUFVUD (see his quotation above).

VII - EXPECTATIONS, THE INDIRECT UTILITY OF MONEY AND TEMPORARY KEYNESIAN EQUILIBRIUM

In the preceding sections we gave ourselves a priori the utility of money as a store of value, which allowed us to ignore the linking between present and future periods through accumulation, as well as the role of future expectations in present equilibria, though these are evidently very important themes of Keynesian analysis.

So we shall study here an economy where future expectations (about prices and constraints) are uncertain (which is more realistic than certain expectations), and money links successive periods as the only store of value.

We will explicit formally how expectations determine the indirect utility, and how the resulting K-equilibrium will be affected (as before, to simplify, prices will be fixed in the first period).

1/ THE ECONOMY

We shall consider a two-period exchange economy. There are n agents, $i = 1 \dots n$. Each one has a utility function over his two-period consumption streams of the Von Neumann-Morgenstern type :

$$U_i \left[\omega_{i1} + z_{i1} , \omega_{i2} + z_{i2}\right]$$

As we want to concentrate on "market uncertainty" rather than "individual uncertainty", we will assume future endowments ω_{i2} known with certainty. Each individual will choose his actions so as to maximize his expected utility with respect to his expectations.

Expectations

Each individual has to forecast prices for the second period p_2 as well as perceived constraints on the goods he will trade $\bar{\bar{z}}_{i2}$ These parameters are not forecasted with certainty but the individual holds a subjective probability distribution on them. This distribution should depend on all information available to the individual in period 1 (past and present prices, past and present perceived constraints, other information variables, ...). Since the "past" as well as the prices in period 1 are given and we want to emphasize especially the importance of present perceived constraints, we shall make the probability distribution explicitly dependent upon the agent's first period perceived constraints :

$$\Psi_i \left[p_2 , \bar{\bar{z}}_{i2} \mid \bar{\bar{z}}_{i1}\right]$$

This probability distribution will be assumed to depend continuously upon its argument $\bar{\bar{z}}_{i1}$ (the set of today's perceived constraints).

2/ THE INDIRECT UTILITY OF MONEY

We are interested in deriving the actions of each agent in the first period, i.e. his effective demands for goods (and desired holding of money).

The most direct way for that would be to compare directly the expected utilities of all actions. We shall rather use here a more manageable criterion, an "indirect utility function" (including in particular money as an argument) which will "summarize" the consequences of each action (14).

Since this indirect utility derives in an essential way from anticipated events and decisions in period two, we study these first.

a) The second-period problem

Consider an individual who has consumed $\omega_{i1} + \bar{z}_{i1}$ in the first period, accumulated a quantity of money M_i.

If he faces a price system p_2 , constraints $\bar{\bar{z}}_{i2}$ in the second period, his second period consumption bundle will be the one maximizing his utility subject to the budget equation and the constraints on all markets (here, since the individual has no demand to express on future markets, we need only to know his expected transactions, which are given by the following program).

./...

(14) - We use here the same methods which were developed in the context of the Hicksian temporary equilibrium [26] by GRANDMONT [16].

Maximize $U_i \left[\omega_{i1} + \bar{z}_{i1} , \omega_{i2} + z_{i2}\right]$ subject to :

$$\left\{\begin{array}{ll} \sum_{h=1}^{\ell} p_{h2} \; z_{ih2} \leqslant M_i & \\ \omega_{i2} + z_{i2} \geqslant 0 & \\ z_{ih2} \leqslant \bar{\bar{z}}_{ih2} & h \in D_{i2} \\ z_{ih2} \geqslant \bar{\bar{z}}_{ih2} & h \in S_{i2} \end{array}\right.$$

We call this last set $\gamma_{i2} \left[p_2 , \bar{\bar{z}}_{i2} , M_i\right]$

The result of this optimization is for each anticipated $(p_2 , \bar{\bar{z}}_{i2})$ an optimal expected vector of consumption $\omega_{i2} + z^*_{i2}$:

$$z^*_{i2} = z^*_{i2} \left[\omega_{i1} + \bar{z}_{i1} , p_2 , \bar{\bar{z}}_{i2} , M_i\right]$$

And an optimal level of utility :

$$U^*_i \left[\omega_{i1} + \bar{z}_{i1} , M_i , p_2 , \bar{\bar{z}}_{i2}\right] = U_i \left[\omega_{i1} + \bar{z}_{i1} , \omega_{i2} + z^*_{i2}\right]$$

As written this maximal utility evidently depends on money holdings and second period anticipated prices and constraints.

b) The indirect utility function

So for each first-period action $(\omega_{i1} + z_{i1} , M_i)$ and each anticipation $(p_2 , \bar{\bar{z}}_{i2})$, the individual can determine his level of utility (given his best action in the second period) :

$$U^*_i \left[\omega_{i1} + z_{i1} , M_i , p_2 , \bar{\bar{z}}_{i2}\right]$$

./...

So, the expected utility of an action as viewed from the first period, is simply the expectation of the above utility with respect to the probabilistic beliefs of the individual.

$$\int_{p_2, \bar{\bar{z}}_{i2}} U_i^* \left[\omega_{i1} + z_{i1} , M_i , p_2 , \bar{\bar{z}}_{i2}\right] d\,\Psi_i \left[p_2 , \bar{\bar{z}}_{i2} \mid \bar{\bar{z}}_{i1}\right] =$$

$$V_i \left[\omega_{i1} + z_{i1} , M_i \mid \bar{\bar{z}}_{i1}\right]$$

This is the indirect utility function. Money is now one of the arguments, together with first-period consumption.

But also most importantly this indirect utility function depends upon anticipated prices and constraints, and thus upon today's perceived constraints [15]. This dependence is likely to increase the instability of multipliers.

For example, unemployment today will cause anticipations of future restrictions on selling of labor, and increase the indirect utility of money ("precautionary motive") so that savings will be relatively greater with unemployment (thus reinforcing the deflationary tendencies).

Conversely, if there has been inflation and constraints on buying, the indirect utility of money will be very low, and people will try to get rid of it ("flight from money"). This will accentuate the inflationary demand for goods.

3/ TEMPORARY K-EQUILIBRIUM

With the help of the indirect utility function, we can now derive the effective demands in the first period, in much the same way as they were in section III.

./...

---

(15) - So we see that the indirect utility function we used throughout implicity implied "fixed expectations" (i.e. Ψ_i independant of $\bar{\bar{z}}_{i1}$).

Equilibrium is also defined almost identically : a temporary K-equilibrium will be a set of $\tilde{z}_{ih}$, $\bar{\bar{z}}_{ih}$, $\bar{z}_{ih}$ such that [16] :

. $\tilde{z}_{ih}$ results from the maximization of $V_i \left[\omega_i + z_i , M_i \mid \bar{\bar{z}}_i\right]$
over the set $\gamma_{ih}\left[p , \bar{\bar{z}}_i\right]$

. $\bar{\bar{z}}_{ih} = G_{ih}\left[\tilde{z}_{1h} , \ldots , \tilde{z}_{nh}\right]$

. $\bar{z}_{ih} = F_{ih}\left[\tilde{z}_{1h} , \ldots , \tilde{z}_{nh}\right]$

It can be viewed again usefully as a fixed point of the following recursive tatonnement process : at time t-1 individuals have expressed effective demands $\tilde{z}_{ih}$ (t - 1), from which result perceived constraints

$$\bar{\bar{z}}_{ih}(t-1) = G_{ih}\left[\tilde{z}_{1h}(t-1) , \ldots , \tilde{z}_{nh}(t-1)\right]$$

Effective demands in the following "round" $\tilde{z}_{ih}(t)$ will result from these perceived constraints through the following programs :

Maximize $V_i\left[\omega_i + z_i , M_i \mid \bar{\bar{z}}_i(t-1)\right]$ subject to :

$$\begin{cases} p\, z_i + M_i \leqslant \bar{M}_i & & \\ \omega_i + z_i \geqslant 0 & M_i \geqslant 0 & \\ z_{ih'} \leqslant \bar{\bar{z}}_{ih'}(t-1) & h' \in D_i & h' \neq h \\ z_{ih'} \geqslant \bar{\bar{z}}_{ih'}(t-1) & h' \in S_i & h' \neq h \end{cases}$$

./...

(16) - We skip here subscript 1 since everything pertains to the first period.

4/ EXISTENCE OF A TEMPORARY K-EQUILIBRIUM

Clearly, a temporary K-equilibrium will exist if the mapping $\tilde{z}_{ih}(t-1) \to \tilde{z}_{ih}(t)$ just defined above has a fixed point. From section IV on K-equilibrium, we know that such a fixed point will exist if the indirect utility functions $V_i\left[\omega_{i1} + z_{i1}, M_i \mid \bar{\bar{z}}_{i1}\right]$ are :

- continuous in $z_{i1}, M_i, \bar{\bar{z}}_{i1}$

- concave in z_{i1}, M_i

For that we need a bit more of assumptions on utilities and expectations :

- $U_i\left[\omega_{i1} + z_{i1}, \omega_{i2} + z_{i2}\right]$ is continuous and concave in its arguments.

- The mapping $\Psi_i\left[p_2, \bar{\bar{z}}_{i2} \mid \bar{\bar{z}}_{i1}\right]$ from the set of first-period constraints to the set of probability measures over second period prices and constraints is continuous with respect to the topology of weak convergence of probability measures (17).

- No price is expected to be zero in the second period (the support of the corresponding probability measure belongs to the interior of the positive orthant).

We can now prove the above properties for each V_i .

a) Concavity

Consider two couples (dropping the subscript i) : (x'_1, M') and (x''_1, M'') and a given $\lambda \in [0, 1]$. Let :

$$\begin{cases} x_1 = \lambda x'_1 + (1-\lambda) x''_1 \\ M = \lambda M' + (1-\lambda) M'' \end{cases}$$

./...

(17) - For some on this assumption of "continuity of expectations", see GRANDMONT [16].

We want to show that :

$$V_i \left[x_1 , M \mid \bar{\bar{z}}_1\right] \geq \lambda V_i \left[x'_1 , M' \mid \bar{\bar{z}}_1\right] + (1 - \lambda) V_i \left[x''_1 , M'' \mid \bar{\bar{z}}_1\right]$$

α) First fix $p_2 , \bar{\bar{z}}_2$

We have :

$$\begin{cases} U^*_i \left[x'_1 , M' , p_2 , \bar{\bar{z}}_2\right] = U_i \left[x'_1 , x'^*_2\right] \\ \\ U^*_i \left[x''_1 , M'' , p_2 , \bar{\bar{z}}_2\right] = U_i \left[x''_1 , x''^*_2\right] \end{cases}$$

As easily checked,

$$\lambda x'^*_2 + (1 - \lambda) x''^*_2 \in \gamma_{i2} \left[p_2 , \bar{\bar{z}}_2 , M\right]$$

Hence

$$U^*_i \left[x_1 , M , p_2 , \bar{\bar{z}}_2\right] \geq U_i \left[x_1 , \lambda x'^*_2 + (1 - \lambda) x''^*_2\right] \geq$$

$$\lambda U^*_i \left[x'_1 , x''^*_2\right] + (1 - \lambda) U_i \left[x''_1 , x''^*_2\right]$$

(the last inequality following from the concavity of U_i).

β) Hence we have shown : for each $p_2 , \bar{\bar{z}}_2$

$$U^*_i \left[x_1 , M , p_2 , \bar{\bar{z}}_2\right] \geq$$

$$\lambda U^*_i \left[x'_1 , M' , p_2 , \bar{\bar{z}}_2\right] + (1 - \lambda) U^*_i \left[x''_1 , M'' , p_2 , \bar{\bar{z}}_2\right]$$

taking the expectation of both sides with respect to the probability distribution $\Psi_i \left[p_2 , \bar{\bar{z}}_2 \mid \bar{\bar{z}}_1\right]$ we obtain the desired result.

Q.E.D.

./...

b) Continuity

With second-period prices strictly positive, the set

$$\gamma_{i2}\left[p_2 , \bar{\bar{z}}_{i2} , M_i\right]$$

is continuous in its arguments [18].

Thus, by the theorem of maximum, the functions

$$U_i^{*}\left[\omega_{i1} + z_{i1} , M_i , p_2 , \bar{z}_{i2}\right]$$

are continuous in their arguments.

Since, in addition, expectations

$$\Psi_i\left[p_2 , \bar{\bar{z}}_{i2} \mid \bar{\bar{z}}_{i1}\right]$$

are continuous, continuity of the function V_i in its arguments follows from Theorem A-3, section 5, in GRANDMONT [17].

So all V_i's satisfy the concavity and continuity assumptions, and a temporary K-equilibrium exists.

Q.E.D.

---

(18) - See BENASSY [4], appendix, where the proof, too long to appear here, was taken from an early unpublished version of DREZE [14].

CONCLUSION

As we saw, the use in a formalized model of the concepts of effective demand and quantity adjustment enriches considerably the traditional neoclassical theory since we can describe with them a decentralized economy functioning at disequilibrium prices. Phenomena like involuntary unemployment, multiplier effects, etc... appear, which make this approach particularly well adapted for an integration of micro and macroeconomic theories.

The equilibrium concept obtained, K-equilibrium, generalizes the traditional notion of Keynesian equilibrium ; it contains notably as particular cases Walrasian or monopolistic equilibria (19).

We find also in our model a particularly important result of Keynesian analysis (20) : in a monetary economy in disequilibrium, signals transmitted under the form of effective demands by the agents give a false idea of the actual exchange possibilities in the economy. The result is the existence of some equilibria (notably "multiplier" equilibria) where the level of exchanges and economic activity is "artificially" depressed, even taking into account the "wrong" exchange rates.

Correlatively, and in a more "dynamic" view, we see that contrarily to the usual price adjustments, quantity adjustments have rather disequilibrating effects, especially if we take quantity expectations into account. These expectations play themselves a very important role in the determination of K-equilibria, which we "summarized" in the indirect utility of money (and it is easily seen that the same methods would apply for any other stock or store of value). Here an explicitly dynamic stock-flow analysis would be particularly desirable, and should be a subject for future research.

---

(19) - See BENASSY [4] [7].

(20) - Cf. KEYNES [28] (Ch. 16), CLOWER [9] [11] LEIJONHUFVUD [30] [31].

APPENDIX

As is the case for tatonnement processes, the ones we presented in the text can describe observable states of the economy at equilibrium points only (i.e. in our K-equilibria). This is due to the fact that we treated all markets symetrically, notably from the information's point of view (as we shall see, everything happens somehow as if each market was the first visited).

In the contrary, if we want to be able to follow the movement of the system in time (i.e. describe a non-tatonnement process), we must take into account the fact that in reality markets are visited sequentially. As it would be too heavy for our purpose to formalize the choice of the order of visit of markets by individuals, we shall assume that this order of visit is given a priori.

1/ THE INSTITUTIONAL FRAMEWORK

As before the analysis will hold in a monetary economy with ℓ markets where each non-monetary good ($h = 1 \dots \ell$) is exchanged against money.

Since we are in a non-tatonnement model, time will consist in a sequence of trading periods, or "market days", indexed by t, during which transactions do actually take place on these ℓ markets.

At the beginning of a period t, each trader i receives a constant endowment of non-monetary goods (ω_i) and carries the quantity of money he held at the end of the previous period :

$$\bar{M}_i (t) = \bar{M}_i (t-1) - \sum_{h=1}^{\ell} p_h \bar{z}_{ih} (t-1)$$

./...

Then each trader visits the ℓ markets in an a priori given order. To simplify the notations, we shall take the ordering of the goods and the order of visit of markets to be the same, i.e. :

$h' > h \iff$ Market h' is visited after market h

$h' < h \iff$ Market h' is visited before market h

On each market h trader i expresses an effective demand $\tilde{z}_{ih}$ (t).

The exchange process on a particular market h yields transactions and perceived constraints in exactly the same way as described in section II :

$$\bar{z}_{ih}(t) = F_{ih}\left[\tilde{z}_{1h}(t), \ldots, \tilde{z}_{nh}(t)\right]$$

$$\bar{\bar{z}}_{ih}(t) = G_{ih}\left[\tilde{z}_{1h}(t), \ldots, \tilde{z}_{nh}(t)\right]$$

We now turn to the determination of effective demands on each of these markets.

2/ EFFECTIVE DEMANDS

Our definition of effective demand has now to take into account the sequentiality of markets, and the corresponding accumulation of information :

The effective demand of trader i on market h is the exchange determined by maximizing his utility, taking into account exchanges already realized in past markets, and expected constraints on future exchanges.

So assume individual i has already realized transactions $\bar{z}_{ih'}$ (t) on markets visited before h $(h' < h)$, and expects constraints $\bar{\bar{z}}^{e}_{ih'}$ (t) on markets he will visit afterwards $(h' > h)$.

./...

Accordingly to our definition, effective demand $\tilde{z}_{ih}$ (t) will be the hth component of the optimum vector of the following program :

Maximize $\quad U_i \left[\omega_i + z_i \, , M_i\right] \qquad$ subject to

$$\begin{cases} p z_i + M_i \leqslant \bar{M}_i(t) & & & \boxed{1} \\ \omega_i + z_i \geqslant 0 & M_i \geqslant 0 & & \boxed{2} \\ z_{ih'} = \bar{z}_{ih'}(t) & h' < h & & \boxed{3} \\ z_{ih'} \leqslant \bar{\bar{z}}^{e}_{ih'}(t) & h' > h & h' \in D_i & \boxed{4} \\ z_{ih'} \geqslant \bar{\bar{z}}^{e}_{ih'}(t) & h' > h & h' \in S_i & \boxed{4'} \\ M_{ih'} \geqslant 0 \;^{(21)} & h' \geqslant h & & \boxed{5} \end{cases}$$

Three main differences can be noted with the tatonnement version of effective demand seen in section III :

a) First, we notice the apparition of transactions constraints (5), which express that the individual never plans to hold a negative quantity of money after a transaction. This type of constraint, of the same nature than CLOWER's [10] well-known expenditure constraint, appears as soon as the hypothesis of simultaneous exchange on all markets is abandoned (22).

./...

(21) - With $M_{ih'} = \bar{M}_i(t) - \sum\limits_{h'' \leqslant h'} p_{h''} z_{ih''}$

$M_{ih'}$ is the amount of money held by i after transacting on market h'.

(22) - For a formal treatment of this constraint in a general equilibrium framework, see for example GRANDMONT-YOUNES [19] [20].

b) We must remark that the constraints on future markets taken into account in constraints (4) and (4'), the $\bar{\bar{z}}^{e}_{ih'}$ (t) are expected, or ex-ante constraints. They should not be mixed with the $\bar{\bar{z}}_{ih'}$ (t) , or ex-post constraints, which arise once the market has been held and effective demands expressed, as described in section II :

$$\bar{\bar{z}}_{ih'}(t) = G_{ih'}\left[\tilde{z}_{1h'}(t), \ldots, \tilde{z}_{nh'}(t)\right]$$

In the tatonnement models we could somehow collapse the two concepts, since at a K-equilibrium ex-ante and ex-post constraints are the same. In the non-tatonnement process, we will have to specify more precisely how the ex-ante constraints are formed through expectations (23).

c) Finally, we see that the individual actually uses the information obtained on past markets by taking into account realized transactions (and constraints) on these markets instead of expected, or ex-ante constraints.

So we see that the symetrized effective demand definition, which took only into account ex-ante constraints, was expressed as if each market was the first visited.

3/ THE NON-TATONNEMENT PROCESS AND EQUILIBRIUM

We are now almost ready to describe the non-tatonnement exchange process in time, i.e. to specify what will be the effective demands at time t $\tilde{z}_{ih}(t)$ provided we know the effective demands expressed in previous periods $\tilde{z}_{ih}(\tau)$ $(\tau < t)$

We still have to specify how the ex-ante constraints are formed, i.e. the expectations pattern for trading constraints. Let us start with a very simple and common pattern :

$$\bar{\bar{z}}^{e}_{ih}(t) = \bar{\bar{z}}_{ih}(t-1)$$

./...

(23) - These points have been emphasized by LEIJONHUFVUD [31].

The ex-ante constraint for period t is expected to be the same as the one observed ex-post in $t - 1$.

Holdings of money and effective demands will be determined sequentially by the following recursive relations :

$$\bar{M}_i\ (t) = \bar{M}_i\ (t - 1) - \sum_{h=1}^{\ell} p_h\ \bar{z}_{ih}\ (t - 1)$$

$\tilde{z}_{ih}$ (t) is the hth component of the optimum vector of the program :

Maximize $U_i \left[\omega_i + z_i\ ,\ M_i\right]$ s.t.

$$\left\{\begin{array}{llll}
pz_i + M_i \leqslant \bar{M}_i\ (t) & & & \boxed{1} \\
\omega_i + z_i \geqslant 0 & M_i \geqslant 0 & & \boxed{2} \\
z_{ih'} = \bar{z}_{ih'}\ (t) & h' < h & & \boxed{3} \\
z_{ih'} \leqslant \bar{\bar{z}}_{ih'}\ (t - 1) & h' > h & h' \in D_i & \boxed{4} \\
z_{ih'} \geqslant \bar{\bar{z}}_{ih'}\ (t - 1) & h' > h & h' \in S_i & \boxed{4'} \\
M_{ih'} \geqslant 0 & h' \geqslant h & & \boxed{5}
\end{array}\right.$$

We see that demands will be first determined on market $h = 1$, then $h = 2 \ldots\ h = \ell$ (because of constraints 3).

We recognize in this process a generalization of the well-known Keynesian dynamic multiplier :

$$\left\{\begin{array}{l}
C_t = c\ Y_{t-1} \\
Y_t = c_t + \bar{I}_t
\end{array}\right.$$

./...

An equilibrium will be a self-reproducing state of this recursive process (and the system will actually converge towards it if it is stable) (24).

As it is easy to verify, at equilibrium the vector of transactions $\bar{z}_i$ and the holdings of money $\bar{M}_i$ are the solutions of the following program :

$$\text{Maximize } U_i \left[\omega_i + z_i , M_i\right] \qquad \text{s.t.}$$

$$\begin{cases} pz_i + M_i \leq \bar{M}_i & \\ \omega_i + z_i \geq 0 & M_i \geq 0 \\ z_{ih} \leq \bar{\bar{z}}_{ih} & h \in D_i \\ z_{ih} \geq \bar{\bar{z}}_{ih} & h \in S_i \\ M_{ih'} \geq 0 & \forall\, h' \end{cases}$$

So that all what we said about the inefficiency of equilibria in section V still holds.

4/ REMARKS

a) The expectations pattern $\bar{\bar{z}}^e_{ih}\ (t) = \bar{\bar{z}}_{ih}\ (t-1)$ is evidently far too simplistic. We can use without changing the analysis more general ones, like :

$$\bar{\bar{z}}^e_{ih}\ (t) = \Psi_{ih} \left[\bar{\bar{z}}_{ih}\ (t-1)\ , \ldots ,\ \bar{\bar{z}}_{ih}\ (t-T)\ , \ldots\right]$$

./...

(24) - The existence proof would be totally similar to the one in section IV, and is thus omitted.

These will yield permanent-income type effects in the dynamic process. Equilibria obtained will be the same, provided

$$\Psi_{ih}\left[\alpha , \ldots , \alpha \ldots\right] = \alpha$$

b) The assumption of an identical order of visit of markets for all individuals is not so restrictive as it seems : an individual needs not to visit all of them, and thus we can describe any realistic situation where an arbitrary order of visit is given for each trader by relabeling adequately markets and goods.

However this is still too strong since not only the order but also the frequency of visits to markets should be parameters of choice.

B I B L I O G R A P H Y

[1] ARROW, K.J. and HAHN, F.H. [1971], General Competitive Analysis, Holden Day, San Francisco, 1971.

[2] BARRO, R.J. and GROSSMAN, H.I. [1971], "A General Disequilibrium Model of Income and Employment", American Economic Review, March 1971.

[3] BARRO R.J. and GROSSMAN H.I. [1974] "Suppressed inflation and the supply multiplier" Review of Economic Studies january 1974.

[4] BENASSY J.P. [1973], "Disequilibrium Theory", unpublished Ph. D. Thesis, Working paper n° 185, C R M S U.C. Berkeley, june 1973.

[5] BENASSY J.P. [1973] "Théroie Néokeynésienne du déséquilibre dans une économie monétaire" CEPREMAP Discussion paper, to appear in "Cahiers du séminaire d'Econométrie C.N.R.S." (1974).

[6] BENASSY J.P. [1974] "Disequilibrium exchange in Barter and Monetary economies" CEPREMAP Discussion paper February 1974, to appear in Economic Inquiry.

[7] BENASSY J.P. [1974] "The disequilibrium approach to monopolistic price setting and General Monopolistic equilibrium" CEPREMAP Discussion paper, March 1974.

[8] CLOWER, R.W. [1960], "Keynes and the Classics : A Dynamical Perspective", Quarterly Journal of Economics, May 1960.

[9] CLOWER, R.W. [1965], "The Keynesian Counterrevolution : A Theoretical Appraisal", in HAHN, R.H. and BRECHLING, [23]

[10] CLOWER, R.W. [1967], "A Reconsideration of the Micro-foundations of Monetary Theory", Western Economic Journal, December 1967.

[11] CLOWER, R.W. [1971], "Theoretical Foundations of Monetary Policy" in Monetary Theory and Monetary Policy in 1970's, edited by CLAYTON, GILBERT, SEDGWICK, Oxford University Press, 1971.

.../...

[12] DEBREU, G. [1959], Theory of Value, Wiley, New York, 1959.

[13] DREZE, J. (Ed.) [1973 a], Allocation under Uncertainty, Equilibrium and Optimality, Macmillan, 1973.

[14] DREZE, J. [1973 b], "Existence of an equilibrium under price rigidity and quantity rationing", CORE Discussion Paper, University of Louvain, August 1973.

[15] GLUSTOFF, E. [1968], "On the existence of aKeynesian equilibrium", Review of Economic Studies, 1968.

[16] GRANDMONT, J.M. [1971], "Short-run Equilibrium Analysis in a Monetary Economy", CEPREMAP Discussion Paper, February 1971, to appear in DREZE [13].

[17] GRANDMONT, J.M. [1972], "Continuity Properties of a Von Neumann-Morgenstern Utility", Journal of Economic Theory, February 1972.

[18] GRANDMONT, J.M. and LAROQUE, G. [1973], "Equilibres Temporaires Keynésiens", CEPREMAP Discussion Paper, November 1973.

[19] GRANDMONT, J.M. and YOUNES, Y. [1972], "On the Role of Money and the Existence of a Monetary Equilibrium", Review of Economic Studies, July 1972.

[20] GRANDMONT, J.M. and YOUNES, Y. [1973], "On the Efficiency of a Monetary Equilibrium", Review of Economic Studies, April 1973.

[21] GROSSMAN, H.I. [1971], "Money, Interest and Prices in Market Disequilibrium", Journal of Political Economy, Septembre 1971.

[22] GROSSMAN, H.I. [1972], "A Choice-theoretic Model of an Income Investment Accelerator", American Economic Review, September 1972.

[23] HAHN, F.H. and BRECHLING, F.P.R. [1965], eds, The Theory of Interest Rates, Proceedings of an I.E.A. Conférence, Macmillan, London, 1965.

[24] HANSEN, B. [1951], A Study in the Theory of Inflation, Allen and Unwin, London, 1951.

[25] HANSEN, B. [1970], A Survey of General Equilibrium Systems, McGraw Hill, New York, 1970.

[26] HICKS, J.R. [1946], Value and Capital (second edition), Oxford University Press, London, 1946.

[27] HICKS, J.R. [1965], Capital and Growth, Oxford University Press, 1965.

[28] KEYNES, J.M. [1936], The General Theory of Employment, Interest and Money, Macmillan, London, 1936.

[29] KORNAI, J. [1971], Antiequilibrium : On Economic Systems Theory and the Tasks of Research, North Holland, Amsterdam, 1971.

[30] LEIJONHUFVUD, A. [1968], On Keynesian Economics and the Economics of Keynes, Oxford University Press, London, 1968.

[31] LEIJONHUFVUD, A. [1973], "Effective Demand Failures", The Swedish Journal of Economics, March 1973.

[32] MARX, K. [1909], "Capital : A Critique of Political Economy", Charles H. Kerr and Company, Chicago, 1909.

[33] MARX, K. [1952], Theories of Surplus Value, International Publishers, New York, 1952.

[34] PATINKIN, D. [1965], Money, Interest and Prices, (second edition), Harper and Row, New York, 1965.

[35] ROBINSON, J. [1942], An Essay on Marxian Economics, Macmillan, London, 1942.

[36] SOLOW, R.M. and STIGLITZ, J.E. [1968] "Output, Employment and Wages in the Short run" Quarterly Journal of Economics November 1968.

[37] YOUNES, Y. [1970], "Sur une notion d'équilibre utilisable dans le cas où les agents économiques ne sont pas assurés de la compatibilité de leurs plans", Contribution to the Seminaire Roy-Malinvaud, January 1970.

[38] YOUNES, Y. [1973], "On the Role of money in the process of exchange and the existence of a non-Walrasian equilibrium", MSSB Working Paper, Berkeley, July 1973.